



INTRODUCTION

What is good mathematics instruction? My first year teaching, I thought it was about teaching all 150 standards, in a nicely structured, step-by-step way, so that my students were prepared for the next grade. Luckily, I had a math coach who asked me, “Is the goal of your lesson to teach arithmetic or mathematics?” That has always stuck with me. She coached me that year to view mathematics as the art of problem solving. It goes so far beyond correctly operating on numbers and arriving at a correct answer. It is an activity that involves thinking, reasoning, discussing, justifying, and proving. She coached me to understand that, through this problem-solving lens, I could teach those 150 standards, but in ways that were not so scaffolded, not so obvious, and not so boring. If you’ve picked up this book, you likely already know all of this. The danger, of course, is that in moving instruction to a remote setting, we have to be sure that our primary goal is to stay true to what mathematics is really about and not reduce it to teaching arithmetic.

What Is Good Mathematics Instruction?

Before we can possibly consider what rich mathematics instruction looks like when done remotely or online, we first have to ensure a common understanding of what rich mathematics instruction is, period. At its core, it is about developing mathematicians—people who see patterns, are curious about those patterns, and wonder if there is order to those patterns. Mathematicians think flexibly and creatively to determine solutions to questions. When their solution is proven incorrect or only works for a subset of situations, they become engrossed in determining why and how and work tirelessly thinking about the problem for long periods of time. Frustrations come and go, but are short-lived, and we call those “productive struggle.” There is nothing like the joy of working in the productive struggle zone and then discovering something new. It brings a sense of euphoria to the mathematician and gives them the motivation to continue their quest of finding patterns in their world, and making sense of those patterns.

How can we as educators empower every student to feel like a mathematician? What are the tenets of this rich instruction that we can carry from the face-to-face world into the realm of distance learning? There are a number of fundamental non-negotiables that research agrees encompass rich mathematics instruction, regardless of the venue in which it occurs. From the student mathematician’s point of view, these non-negotiables include the five process standards—more recently articulated in eight numbered standards for mathematical practice (SFMPs)—that characterize “doing” mathematics, and from the educator’s point of view, these non-negotiables include the eight Mathematical Teaching Practices (MTPs) (see Figure i.1 on the facing page).

Another non-negotiable in rich mathematics teaching is attention to access and equity. The National Council of Teachers of Mathematics (NCTM, 2014a) describes access and equity in mathematics education in their position statement as follows:

Creating, supporting, and sustaining a culture of access and equity require being responsive to students’ backgrounds, experiences, cultural perspectives, traditions, and knowledge when designing and implementing a mathematics program and assessing its effectiveness. Acknowledging and addressing factors that contribute to

differential outcomes among groups of students are critical to ensuring that all students routinely have opportunities to experience high-quality mathematics instruction, learn challenging mathematics content, and receive the support necessary to be successful. Addressing equity and access includes both ensuring that all students attain mathematics proficiency and increasing the numbers of students from all racial, ethnic, linguistic, gender, and socioeconomic groups who attain the highest levels of mathematics achievement.

Figure i.1

Rich Mathematics Instruction as Defined by NCTM’s Process and Practice Standards and the Eight Mathematical Teaching Practices

NCTM PROCESS AND PRACTICE STANDARDS	MATHEMATICAL TEACHING PRACTICES
<ul style="list-style-type: none"> • Problem Solving SFMP 1. Make sense of problems and persevere in solving them. SFMP 5. Use appropriate tools strategically. • Reasoning and Proof SFMP 2. Reason abstractly and quantitatively. SFMP 3. Construct viable arguments and critique the reasoning of others. SFMP 8. Look for and express regularity in repeated reasoning. • Communications SFMP 3. Construct viable arguments and critique the reasoning of others. • Connections SFMP 6. Attend to precision. SFMP 7. Look for and make use of structure. • Representations SFMP 4. Model with mathematics. 	<ol style="list-style-type: none"> 1. Establish mathematics goals to focus learning. 2. Implement tasks that promote reasoning and problem solving. 3. Use and connect mathematical representations. 4. Facilitate meaningful mathematical discourse. 5. Pose purposeful questions. 6. Build procedural fluency from conceptual understanding. 7. Support productive struggle in learning mathematics. 8. Elicit and use evidence of student thinking.

Source: Adapted from *Principles and standards for school mathematics* (NCTM, 2000), *Common Core State Standards for Mathematics* (National Governors Association Center for Best Practices and Council of Chief State School Officers, 2010), and *Principles to actions: Ensuring mathematical success for all* (NCTM, 2014).

The National Council of Supervisors of Mathematics (NCSM) and TODOS: Mathematics for All (2016) add to this statement that

a social justice stance interrogates and challenges the roles power, privilege, and oppression play in the current unjust system of mathematics education—and in society as a whole.

With the transition to remote learning comes another lens for considering equity and access, and that is the technology that we use to deliver instruction. Therefore, it is critical that we consider the International Society for Technology in Education (ISTE, n.d.) essential condition: “Robust and reliable access to current and emerging technologies and digital resources, with connectivity for all students, including those with special needs, teachers, staff, and school leaders.”

The guidance, suggestions, and activities in this book deliberately pay heed to and are in service of these fundamentals of good mathematics instruction.

What Does Rich Mathematics Instruction Look Like?

Einstein is known for the following statement: “If I had an hour to solve a problem, I’d spend 55 minutes thinking about the problem and five minutes thinking about solutions.” Polya (1945) gives us timeless advice from almost a century ago that good teachers don’t tell students the mathematics, they ask it. But how is this possible when we have so many standards to teach in a year? By following the principles and practices shown in Figure i.1.

Let’s look at an example from both the teacher’s point of view and the student’s point of view. Imagine that you are teaching a lesson on area. This example explores the area of a triangle, but as you read this, consider the learning progression of this big idea and how it applies to your grade-level content. In elementary school, students count the number of squares in a rectangular array, middle schoolers use formulas to find the area of 2-D figures, and calculus students calculate the area under a curve. In this example, Miss Cimorelli incorporates the math teaching practices in her lesson about the area of a triangle.

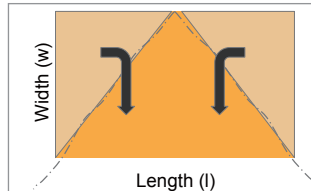
<p>Establish mathematical goals to focus the lesson. (MTP 1)</p>	<p>Goal: Students will explore relationships between triangles and rectangles in order to determine the formula for area of a triangle.</p>
<p>Implement tasks that promote reasoning and problem solving. (MTP 2)</p>	<p>She decided to introduce a rich task that will motivate students to think flexibly as they find relationships between rectangles and triangles. The task required students to consider different cake pans for a cake company to create a triangular cake.</p> <p>This task is open-ended and allows for every student to access the task, regardless of background knowledge of the area of triangles and rectangles. For example, she knew from her preassessment that Kate has very little background knowledge on calculating area of shapes, but she is very skilled at visualizing geometric shapes. Zach is in many ways opposite in that he is quick to solve area problems using standard formulas, but rarely uses geometric models to explain his reasoning.</p>
<p>Support productive struggle in learning mathematics. (MTP 7)</p>	<p>Miss Cimorelli noticed that Zach is frustrated because he can only imagine the cakes cut in one direction, diagonally, and isn't motivated to find other ways. She asked Zach purposeful questions to encourage him to think with curiosity.</p> <div data-bbox="725 993 1032 1191" data-label="Image"> </div>
<p>Pose purposeful questions. (MTP 5)</p>	<p>Miss Cimorelli knew that Zach was interested in the solution, but he needed to learn how to justify his reasoning and to justify it in multiple ways. To do this, she used purposeful questioning to guide him, while not overscaffolding.</p> <p>Miss Cimorelli: Zach, how did you find this example?</p> <p>Zach: I knew the formula and this is how you make it half.</p> <p>Miss Cimorelli: Are there other ways of making $\frac{1}{2}$?</p> <p>Zach: Yeah, but I don't know how.</p> <p>Miss Cimorelli: Since the bakers can use icing as glue, this lets us make as many cuts as we want. What other ways could you cut the rectangle and rearrange the pieces to make triangles?</p>

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Zach: Ohhh, I guess there are lots of ways.

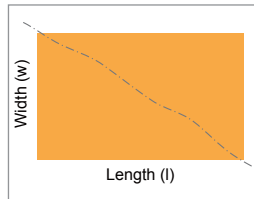
Zach begins another model:



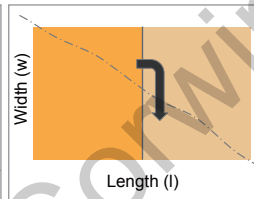
Use evidence of student thinking. (MTP 8)

In preparing for the whole-class discussion, Miss Cimorelli selected three student samples. While there were additional representations available, she knew that by selecting only a few, she could facilitate a deeper conversation about the connections between these three representations.

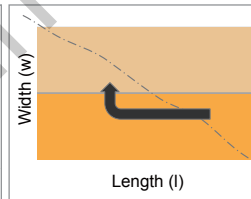
Zach's Thinking



Paige's Thinking



Han's Thinking



Facilitate meaningful mathematical discourse. (MTP 4)

In the vignette that follows, Miss Cimorelli facilitated a whole-class discussion. She knew the importance of asking questions that support student understanding and sense-making.

Use and connect mathematical representations. (MTP 3)

Miss Cimorelli used questions that required students to compare the three models in order to connect the mathematical representations.

Miss Cimorelli: Zach, you mentioned that you knew the formula for area of a triangle. How did that help you to make the cuts on the cake?

Zach: Well, I knew one-half times length times width, so I just cut it in half and you can see that there are two triangles that are exactly the same. *(Zach flipped the other triangle so that it lies on top.)*

Miss Cimorelli: Paige, you found another way of looking at one-half. Can you explain your way to us?

Paige: I cut the rectangle in half first, then showed it was the same as Zach's by flipping the top triangle to cover the space in the lighter triangle. This shows that the area of half of my rectangle is the same as the area of Zach's triangle.

Miss Cimorelli: What do these two have in common?

DeMarco: They are both talking about half of the rectangle is all you need to make that triangle.

Miss Cimorelli: Who can rephrase DeMarco and tell us more?

Yessica: He's right, they are both half, it just depends when you do the half part. In Zach's, he cut it in half at the beginning and made two same triangles, but Paige cut the rectangle in half and then showed that you can see the same triangle with just that much cake.

Miss Cimorelli: Han, can you show us your design?

Han: It's the same as Paige, I just cut it sideways.

Miss Cimorelli: Han said it was the same as Paige. Who can tell us what is the same and what is different?

Melanie: They both used less cake. It's like the cake pan can be half the size and then you move a little chunk of cake to make the triangle, but Zach's has two triangles so it's twice as much cake.

Miss Cimorelli: Let's reconsider that formula $\frac{1}{2}(l \times w)$. What does that mean in each situation?

Zach: So the length times width is how you find the area of a rectangle, and I just took half.

Melanie: But Paige started with just half, half the length and Han started with half the width. So it's like $(\frac{1}{2} \times l) \times w$ or $l \times (\frac{1}{2} \times w)$.

Zach: But those formulas are all the same.

Miss Cimorelli: Indeed, the expressions are equal to one another, but the visuals help us to see how they are equal. For tonight's homework, you are going to draw visuals as you practice the formula.

Build procedural fluency from conceptual understanding. (MTP 6)

Miss Cimorelli assigned students additional practice using one of the three versions of the formula that they discussed in class. As students practiced using the formulas, they were practicing procedural fluency that is based not on a series of letters and numbers, but on the conceptual understanding of $\frac{1}{2}$ that they explored in class.

Let's consider this same task from the student's point of view using the NCTM process standards.

Problem Solving

Students were tasked with exploring a real-world situation—triangular cakes and the way that bakers could cut the cakes to create triangular pieces. They had many tools available, including paper, cake pans, rulers, scissors, and pencils. The problem was open-ended enough to support various ideas, giving each student an opportunity to problem-solve independently, in small groups, and in the whole group.

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Reasoning and Proof	Students explored the conceptual understanding behind the abstract representation, or formula, of the area of a triangle and its relationship to $\frac{1}{2}$ of a rectangle with similar length and width measurements. Through exploration, students were able to reason through various examples to determine how the $\frac{1}{2}$ was preserved in the model and formula.
Communication	Students communicated in their small groups and whole groups as they explored the designs that peers presented. Since students had various background knowledge, they needed to communicate clearly how their visual model matched the abstract model, or formula, or how their visual model matched another student's visual model.
Connections	During the whole-group discussion, students found connections between the three visual representations and three ways of interpreting the abstract formula. They found similarities (all had $\frac{1}{2}$) and differences (the half was subdivided in different ways) between the three visual models in order to make the generalization that the area of a triangle is $\frac{1}{2}$ that of the rectangular cake pan.
Representations	Students were encouraged to create many representations. There were several ways that a rectangle could be cut into two equal triangles, and each representation could be compared to others in order to make generalizations about the abstract representation or formula.

INSTRUCTIONAL MOVES THAT SUPPORT RICH INSTRUCTION

Ensuring rich mathematics instruction doesn't just happen by magic. It is made up of a thousand smaller choices and micro moves that you make every day to make it a reality, such as the following:

- **Setting, practicing, and reinforcing clear norms:** Teaching mathematics begins with setting up a space to learn. Both physical spaces, such as desks or carpet spots, and social norms are needed in order to create a classroom environment that is inclusive, open to mistakes, and rich with student discourse, all while maintaining a structured and orderly environment. This setup is fundamental to classroom management, and these physical and social spaces need to be identified

and practiced in the remote setting so that students are responsible and accountable for learning mathematics.

- **Connecting mathematical representations:** Teaching mathematics is so much more than showing students how to solve an equation; it is about visualizing, connecting, and modeling problems. An abundance of tools and manipulatives are available to students in face-to-face classrooms that support their conceptual understanding. As students model patterns with manipulatives, they are better able to transfer those models into numbers, and then into generalizations and rules. This crucial transition is needed in the remote classroom and can be accessed using virtual manipulatives as well as household items.
- **Offering daily structure:** Mathematics classrooms have various structures that purposefully engage students in short routines to multiday projects, low to high levels of cognitive demand, and new-to-review concepts. A skillful teacher balances the needs of their learners to include daily routines, whole-class tasks, small-group guided instruction, games, projects, assessments, and homework. These same structures are needed in the remote classroom. Through the use of multiple virtual modalities, students can speak and listen to each other, show and view representations, and even play games together.
- **Making student thinking visible:** When teachers use student work as evidence of understanding, it is required that the teacher first *see* the student's work. In the face-to-face classroom, this is seen when the teacher walks around the classroom and observes students building models, drawing pictures, talking with a partner, writing equations, and even making facial expressions. This observation is far more skilled than ensuring that students are on task; it incorporates many elements of formative assessment and is just as vital in the remote classroom. Many programs and applications bridge the physical distance between teacher and student and provide opportunities for students to upload video and images live, and in real time, so that teachers can watch the learning unfold in its raw, rough-draft form.

- **Practicing meaningful formative and summative assessment:** Teaching mathematics is a never-ending progression of concepts and ideas based on prior mathematical knowledge. Great teachers preassess this knowledge from each student and differentiate activities and lessons to guide students as they progress to new understanding. Along the way, the teacher must assess the students' understanding in order to determine the next set of activities and lessons. This constant assessment and planning is at the heart of great teaching, and it goes far beyond any multiple-choice test. This need for assessment is just as important in the remote classroom, but students have much more agency in how they complete the assessment. Gone are the days of solving simple equations with one-number answers because the technology gives students this access at their fingertips. Gone are the days of every student having the same test because of the immediate transfer speeds over social media. Assessments in the remote classroom use robust, problem-oriented situations that require students to apply basic number sense to rich situations.

Translating Rich Instruction to a Virtual Setting

It can seem daunting to reimagine how the practices, routines, and activities that you have mastered in your face-to-face teaching might work in an online environment. First, we have to think about the transition process and the role technology plays, and then we can have a clearer sense of what rich online math instruction looks like.

USING TECHNOLOGY TO TRANSLATE MATH TO DISTANCE LEARNING

The first step to reimaging instruction to a remote setting is to think through what technology does and doesn't allow us to do. I use the Substitution, Augmentation, Modification, Redefinition (SAMR) model (Puentedura, 2013) when considering how to reimagine a structure. The SAMR model is a framework that identifies *how* technology is used in comparison to *how* the assignment was accomplished without the technology. This framework

uses a continuum from Substitution—meaning that the technology didn't bring anything new to the assignment, just a difference in the modality by which it is submitted or completed—to Redefinition—meaning that the assignment could not have been imagined without the tools offered by the technology. This is a useful framework when considering how to transition assignments from the face-to-face setting to the online setting. Each level of the model defines the level of technology integration (see Figure i.2).

Consider the activity of eliciting background knowledge through the Brain Dump strategy in a face-to-face class. The teacher begins by asking all students what they know about angles and triangles, for example. Students raise their hands and quickly respond with many ideas as the teacher writes them down on the whiteboard. Once the ideas are listed, the teacher then instructs students to work together in small groups to sort the ideas into various categories of their choosing in order to make a web diagram about what they know about triangles. At several times throughout the unit, the students will return to their web diagram and update it. At times, the updates will only need to include written additions, crossing out previous connections and revising them, or complete revisions on new paper. Finally, at the end of the unit, the groups present their web diagrams and engage in a whole-group conversation about how the web diagrams show their understanding of the unit. This activity leverages the advantages of prior knowledge, group work, visual representations, connections between mathematical ideas, whole-group discussion, and revisions. But how do you implement those in a remote classroom, and how can the SAMR model support you in reimagining this activity to be *better* than what is possible in the face-to-face class? Let's structure the transition.

Figure i.2

Sample Activity Using the SAMR Model

SUBSTITUTION	AUGMENTATION	MODIFICATION	REDEFINITION
Uses technology to simply substitute an activity or assignment without changing its function	Uses technology to substitute an activity or assignment but with some functional improvement	Uses technology to make the task better than it could have been accomplished without technology	Uses technology in a way that the task simply could not be accomplished without the technology

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SUBSTITUTION	AUGMENTATION	MODIFICATION	REDEFINITION
<p>Students raise their virtual hand and the teacher uses a document camera to record all the various ideas. Then the students are sent to breakout rooms to create their web diagram using the interactive whiteboard, which allows all students to use a virtual marker and create their diagram.</p>	<p>Students interact using the previous step, but instead of using an interactive whiteboard, they create their web diagram using interactive slides where they can use options like copy/paste to make the web diagram process more efficient. Throughout the unit, students can simply move terms around the slide rather than creating a new slide.</p>	<p>The students do not raise a hand but rather type directly on the shared slide. The teacher gives enough time for students to all type a few ideas. Then the students are sent to breakout rooms with a duplicate of the previous slide as they create the web diagram on their own slide. Simultaneously, they can see the web diagrams of other groups as they evolve in real time. The teacher can also view these web diagrams on the shared slides even when in different breakout rooms.</p> <p>Throughout the unit, students and teachers can view the history of the web diagram to determine how it changed over the course of the unit.</p>	<p>Students interact using the previous step but also elicit comments from around the world by uploading their web diagram to social media. Each time they revise their web diagram, they must attend to the comments posted by other users by either responding to the comments or using the comment in the revision. When the final web diagram is complete, they conduct a live YouTube video and use emojis and comments in real time to edit their presentation.</p>
<p>There is no functional change. The physical hands are virtual hands, the whiteboard is a virtual whiteboard, and the small groups still discuss and draw using microphones and a virtual whiteboard.</p>	<p>There is little functional change that leads to efficient revisions. Tools such as copy and paste, ability to move objects rather than rewriting them, and duplicate objects create an efficient experience while maintaining the same fundamental activity.</p>	<p>Technology is in the forefront, and this task relies on the technology in order to be implemented. Students are no longer waiting their turn to add to the brain dump, they are all simultaneously adding. Small groups are no longer working in isolation, they are</p>	<p>This task is completely redefined in the public space. Social media and live video presentations give students the opportunity to explain their product with a variety of perspectives, not just those of students and teachers at their school or in their community.</p>

SUBSTITUTION	AUGMENTATION	MODIFICATION	REDEFINITION
		able to view the updated progress of all other groups in the class. This task is completely modified to give more students a voice.	

Let's look at a high school geometry example. Consider the proof that the sum of all angles in a triangle is 180 degrees. In a typical face-to-face classroom, students might cut out a triangle, rip off the corners, and realign them to show that they create a straight line, which equals 180 degrees. If the entire class of students completes this activity, the whole class might observe 20 to 30 examples, and even a few nonexamples due to human inaccuracies. This can lead them to developing a proof.

This same activity can be done online, and we can use the SAMR framework to consider *how* the technology is used. The **Substitution** level defines activities that use technology but have no functional change. For example, students can still do the paper-ripping activity, take a snapshot of the product, and upload it to a shared space so that all students can view the 20 to 30 triangle examples. This is classified as substitution because the transition to online learning has no functional change, just the modality by which it is presented.

The **Augmentation** level defines activities that go beyond the Substitution level to add functional change. For example, the ability to copy and paste an image of a triangle in order to create multiple perfectly congruent shapes (without human error) allows the exploration of the activity to become more exact. The copy/paste feature is a great improvement, but it doesn't ultimately change the task.

The **Modification** level defines activities that use technology for significant task redesign. For example, when students explore triangles using geometry software (Geogebra, Geometers sketchpad, or CAD), they can twist, turn, flip, invert, and modify the triangle into infinite possibilities while viewing the change of the angles and consistent sum. This technology gives them the opportunity to view every single possibility imaginable. This provides a significant task redesign.

The **Redefinition** level defines activities that simply cannot be implemented without technology. For example, after geometry students explore the relationship between triangles and the sum of angles, they create a presentation and present it live on YouTube, where other high school students from around the world watch, comment, and use emojis to respond in real time. The presenters use this information to answer questions in real time and clarify any misconceptions. This peer interaction with students around the world offers a unique task redesign that still focuses on the initial goals of the task: that students can explore the relationship between triangles and the sum of their angles and be able to use reasoning to create a proof.

REIMAGINING RICH MATHEMATICS INSTRUCTION ONLINE

With the SAMR model in mind, let's briefly take a look at some initial concrete ways that rich mathematics instruction based on fundamental process or practice standards can look when conducted at a distance (see Figure i.3).

Figure i.3

Sample Ideas for Reimagining Math Instruction at a Distance

Problem Solving: Students should explore meaningful and relatable problems using a variety of strategies to solve and self-assess their understanding of mathematical ideas.	
FAMILIAR USE IN A FACE-TO-FACE CLASSROOM	REIMAGINED USE IN AN ONLINE CLASSROOM
<ul style="list-style-type: none"> • Work independently while recording their thinking on paper. 	<ul style="list-style-type: none"> • Students can turn off sound and microphones to incorporate quiet independent work time.
<ul style="list-style-type: none"> • Problem-solve in small groups to develop strategies. Notebooks are usually used and shown to group members to convey ideas. 	<ul style="list-style-type: none"> • Use interactive slides to share ideas to other groupmates. Students can copy/paste peers' ideas to modify while preserving original strategy.
<ul style="list-style-type: none"> • After the math task discussion, the student can self-assess their understanding of the problem, solutions, and strategies by comparing their notes to those presented. 	<ul style="list-style-type: none"> • Students can, in the moment, self-assess their strategies as they view other groupmates or entirely different groups developing strategies on interactive slides.

Reasoning and Proof: Students should explore the “why” to solutions, make conjectures, and use logical reasoning to determine if an argument is logical by creating examples and nonexamples.

FAMILIAR USE IN A FACE-TO FACE-CLASSROOM	REIMAGINED USE IN AN ONLINE CLASSROOM
<ul style="list-style-type: none"> Students share their reasoning with classmates through partner work and whole-group discussions. 	<ul style="list-style-type: none"> Students can efficiently make duplicate copies of mathematical ideas in order to display patterns that evolve into generalizations.
<ul style="list-style-type: none"> Students can cooperate as a class to create multiple examples and nonexamples to determine generalizations that lead to proofs. 	<ul style="list-style-type: none"> Students can use mathematical software to explore infinite possibilities with the click of a button.

Communication: Students should discuss mathematical ideas, strategies, examples, and nonexamples using both familiar and mathematical language as a way to examine their thinking.

FAMILIAR USE IN A FACE-TO-FACE CLASSROOM	REIMAGINED USE IN AN ONLINE CLASSROOM
<ul style="list-style-type: none"> Partner-share, small-group, and whole-group discussions are facilitated by talking, listening, pointing, visuals, and facial expressions. 	<ul style="list-style-type: none"> The teacher can facilitate breakout rooms with partners or small groups of students who discuss their ideas through multiple modalities: voice, video, chat, and interactive slides.
<ul style="list-style-type: none"> Students take turns speaking. 	<ul style="list-style-type: none"> All students can have a voice simultaneously through interactive slides and chat box while one person is speaking.

Connections: Students should explore connections between problems solved in the math classroom with prior experience and problems in their world. They should also explore how mathematical ideas are connected to one another through various notations, strategies, and representations.

FAMILIAR USE IN A FACE-TO-FACE CLASSROOM	REIMAGINED USE IN AN ONLINE CLASSROOM
<ul style="list-style-type: none"> Teachers prepare images or videos of real-world situations to build prior knowledge. 	<ul style="list-style-type: none"> Students provide images (copy/pasted into interactive slides) of prior knowledge and real-world experiences.
<ul style="list-style-type: none"> Students take turns making connections orally during whole-group discussion. 	<ul style="list-style-type: none"> Students respond through multiple modalities to make connections and can copy/paste strategies next to one another.

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Representation: Students should explore problems that use and create mathematical models and representations and explore relationships as they move from one representation to another.

FAMILIAR USE IN A FACE-TO-FACE CLASSROOM	REIMAGINED USE IN AN ONLINE CLASSROOM
<ul style="list-style-type: none">Handheld manipulatives such as pattern blocks, snap cubes, and counters	<ul style="list-style-type: none">Household manipulatives such as buttons, cereal, and blocksVirtual manipulativesCollaborative manipulatives used in interactive slides
<ul style="list-style-type: none">Drawings in notebooks	<ul style="list-style-type: none">Drawings in notebooks and uploaded onto interactive slides where other students can copy and edit the drawings
<ul style="list-style-type: none">Drawings on posters or chalkboard	<ul style="list-style-type: none">Snapshots of notebook drawings or video of notebook entry shared to the class
<ul style="list-style-type: none">Algorithms and procedures recorded step by step, handwritten in a notebook	<ul style="list-style-type: none">Algorithms and procedures annotated through a prerecorded videoAlgorithms and procedures recorded step by step, handwritten in a notebook, and uploaded via image snapshotEquation notation software used to digitize the steps
<ul style="list-style-type: none">Tables and graphs in calculators	<ul style="list-style-type: none">Collaborative data input and manipulation using interactive spreadsheetsScreenshots of tables and graphs created in virtual manipulatives

The goal of this book is to give you situations that require you to think about good mathematics teaching and learning and purposeful transitions to remote instruction. You will learn the purposeful moves that teachers consider when making the transition, and how they keep mathematics at the heart as they reflect and reimagine their remote instruction.