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1

Fractals

A Metaphor for Constructivism, Patterns, and Perspective

> *Why is geometry often described as cold and dry? One reason lies in its inability to describe the shape of a cloud, a mountain, a coastline, or a tree. Clouds are not spheres, mountains are not cones, coastlines are not circles, and bark is not smooth, nor does lightning travel in a straight line.*
>
> Benoit Mandelbrot (1983, p. 1)

BACKGROUND: WHAT IS A FRACTAL?

Perhaps the most frequently studied and visually appealing principle of chaos theory is the spectacular, computer-generated set of fractals, first conceived by modern mathematician Benoit Mandelbrot (1983). Frustrated with the perfectly regular shapes of Euclidean geometry, Mandelbrot developed a whole new geometry more in tune with the natural world. He called the irregular shapes *fractals*, meaning fragmented and irregular. Trees exhibit a chaotic growth plan. Clouds change constantly. Mountains result from a combination of tectonic forces and erosion processes. Nature evolves in irregular patterns: Bird beaks, canyons, sand, and waves are intricately fashioned by the dynamic forces of growth, evolution, and erosion.

Whether viewed from afar or from a closer perspective, natural irregularities repeat on smaller and smaller scales until they are no longer discernible to the human eye. The delicate shape of a fern or a feather does not behave in straight lines and perfect curves. Viewing patterns within patterns, Mandelbrot (1983) explained:

> Nature exhibits not simply a higher degree but an altogether different level of complexity. The number of distinct scales of length of natural patterns is for all practical purposes infinite. (p. 1)

Nature's patterns are never exact.

The Koch snowflake, well-known to mathematicians, is a fractal shape that was first constructed in 1904 by Helge von Koch, a Swedish mathematician. (To construct a Koch snowflake, see Figure 1.1.) When magnified and rotated, exactly four pieces of the snowflake's edge, known as the Koch curve, yield the entire edge of the snowflake (Figure 1.2). Congruent circles may be drawn around the original triangle and each transformed view of the snowflake as it progresses from simple to complex. Although the snowflake always fits within a finite area, the perimeter of the snowflake is infinite. No piece of string could ever be long enough to fit completely around the snowflake's edge (Devaney, 1992).

The Koch snowflake may be compared to a lake or ocean shoreline when viewed from a variety of different perspectives. Coastlines become increasingly longer as more and more detail is included in the measure. When viewed from an airplane, much of the shoreline detail is smoothed over and lost; however, when viewed from a closer perspective, more details appear, and the perimeter increases. Unless a scale is agreed on, all coastlines are infinite in length (Briggs & Peat, 1989). No wonder reference books give conflicting mileage for the same shoreline. A snail traveling around each tiny sand pebble would crawl farther than a deer would leap! Imagine following the shoreline journey of a microscopic organism.

Mandelbrot (1983) retold a story in which young children are asked how long they think the coastline of the eastern United States is. Children understand immediately that the coastline is as long as they want to make it, and that the perimeter increases as each bay and inlet is measured with increasingly smaller scales. Margaret Wheatley (1994) discussed the futility of searching for precise measures for fractal systems:

> Since there can be no definite measurement, what is important in a fractal landscape is to note the quality of the system—its complexity and distinguishing shapes, and how it differs from other fractals. (p. 129)

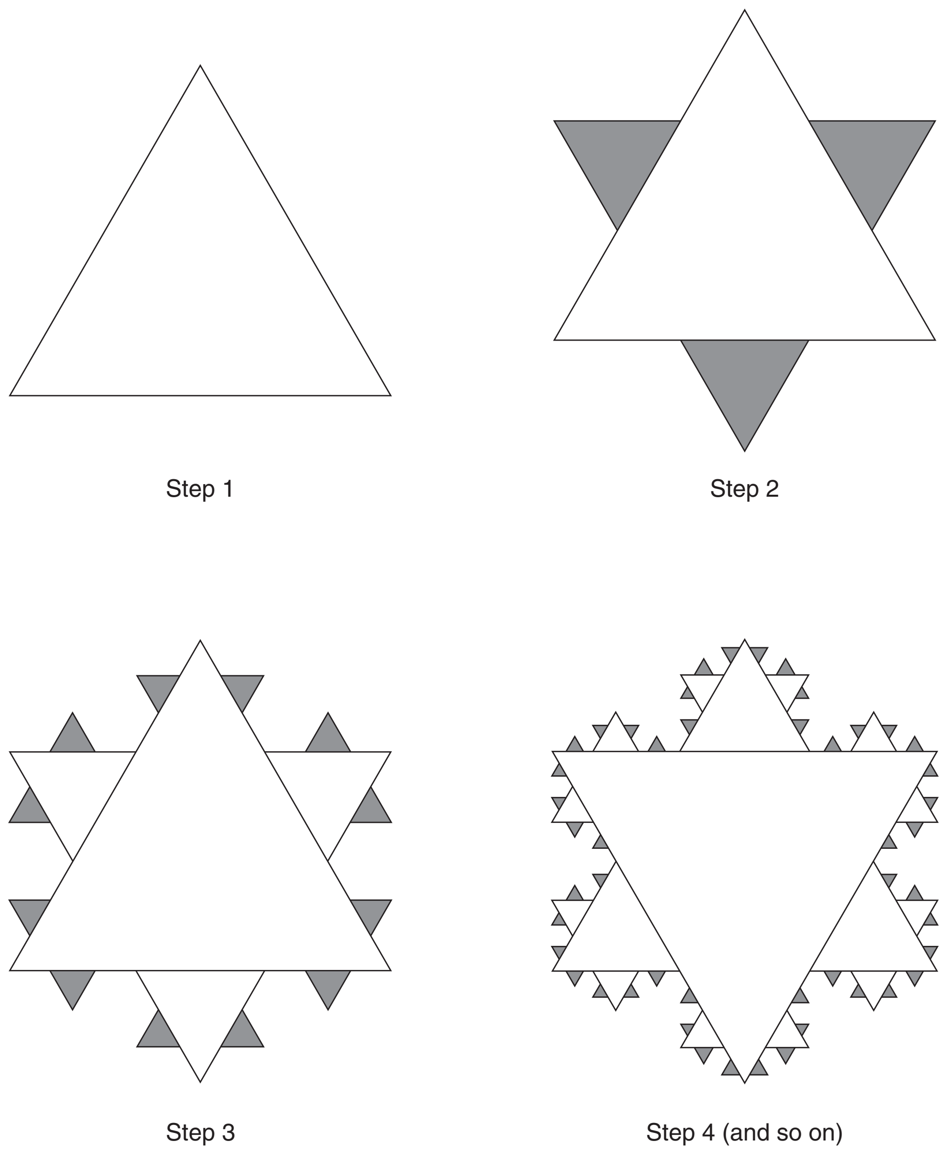
Mandelbrot's coastline question has no final answer.

Mandelbrot (1983) discovered a surprising paradox emerging from chaos theory and fractal geometry. Complex fractals may be generated from a simple mathematical process. In the 1980s, using computer technology, Mandelbrot first viewed the fractal set that has been described as the most beautiful image of modern mathematics (Figure 1.3). A voyage through the Mandelbrot Set reveals finer and finer scales of increasing complexity, sea-horse tails, pinecone spirals, and island molecules all resembling the whole set (Gleick, 1987). The beautiful jewel-like shapes are symbolic of the nonlinear, modern world so full of complexities and seemingly insolvable problems.

If simple leads to complex, could not the reverse be true? Margaret Wheatley and Myron Kellner-Rogers (1996) believed so when they discussed how people seek organization and order in the world, even when chaos is present at the start. "Life is attracted to order—order gained through wandering explorations into new relationships and new possibilities" (p. 6). Perhaps complex problems have simple solutions that remain hidden from view until the time is right for their simple truths to come forth.

Paradoxes such as simple to complex often appear in today's world. Paradoxes enrich people's thinking by providing them with more choices, allowing them to view things differently, and opening up the world for them to generate and create

Directions for Koch Snowflake



Take the middle one-third of each side of an equilateral triangle and attach a new triangle one-third the size. Continue attaching smaller-scale triangles onto the middle third of each new triangle side, resulting in a self-similar and increasingly more intricate snowflake pattern (Gleick, 1987).

Figure 1.1

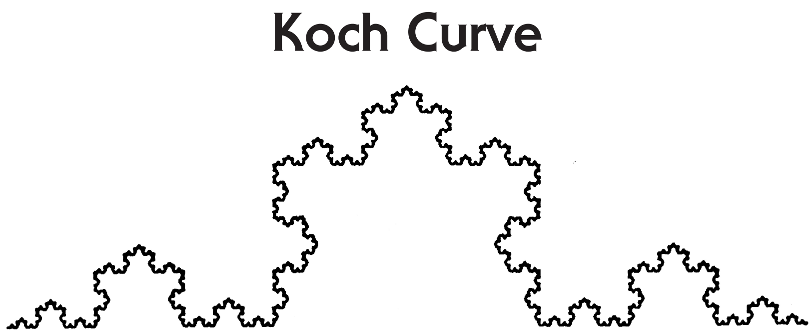


Figure 1.2

their own meaning. John Briggs and F. David Peat (1989) asserted that in the future fractals will undoubtedly reveal more about how chaos hides within regularity and how stability and order can arise out of turbulence and chance. New ways of doing and being may be contained within the old. Unusual and surprising patterns may emerge when least expected. If teachers were to look deeply within themselves for answers, hidden patterns, and universal themes, perhaps they could generate novel purpose and vitality to science education.

IMPLICATIONS OF FRACTALS FOR BRAIN-COMPATIBLE SCIENCE

Three implications of fractals for brain-compatible science are (1) wait for simple truths to reveal greater complexities, (2) construct new meaning from the old, and (3) search for repeating patterns and different perspectives. Parallels exist linking fractal theory to brain-compatible science. Teachers need only to look at the many natural patterns surrounding them to understand the innate ability of the human brain to make connections by constructing new meaning from the old.

Wait for Simple Truths to Reveal Greater Complexities

Wheatley and Kellner-Rogers (1996) raised significant questions about the role of structure. They wondered what people could accomplish if they stopped trying to impose structure on the world and instead found a simpler way of doing things by working with life's natural tendency to organize. Educators might ask this question of themselves as they begin a lesson with their students. Life's events are patterned similarly yet differently each day. Their beautiful, fractal patterns occasionally reveal themselves, often times taking people by surprise. Will people be receptive to the simple truths of life's events when patterns emerge? Perhaps fractals can teach educators something about simplifying the educational process.

Teachers tell students to line up, get it straight, sit up straight, don't get out of line, toe the line, stay on the mark. But where is the line? Where are the mark and the straight? Perhaps they are still back in the age of machines and clockwork precision. Teachers' zest for lesson planning, structuring, and assessing often overshadows the sparks of creative insight that they forget to look for. In a frenzy to get things done and to prepare for tomorrow or next week, teachers rush from one thing to the next, missing out on beautiful moments of fractal simplicity. Perhaps teachers need to slow down, allowing their brains to reveal the simple truths hidden in the complex schedules and endless lists of things to do. Freed from the complexities that get in the way of life's natural tendency to organize, teachers may find a much more creative and energetic form of complexity, one that leads to new discoveries and new understandings about the educational process.

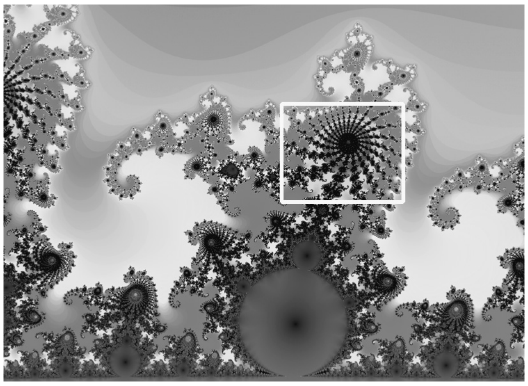
Construct New Meaning From the Old

Geoffrey and Renate Caine, current leaders in brain and learning theory, believe in the brain's constructivist capacity to make patterned connections among the learning task, current experiences, past knowledge, and future behavior. Learning occurs when new information is added to the existing information within the brain and linkages develop between the old and new elements. Students construct their own meaning by reflecting on and integrating new science knowledge with the old. As concepts "construct," broader and more inclusive concepts develop. In one of their 12 mind/brain principles (see Introduction), the Caines (2006) stated that learning involves both conscious and unconscious processes as understanding continues to construct in the brain long after the original learning experience. Teachers need to present material in different ways, using real-life examples to make learning meaningful.

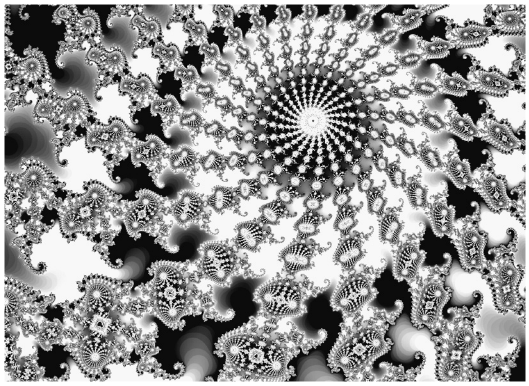
Stephanie Pace Marshall (1999) compared the old educational paradigm with the new in her 13 "principles for the new story of learning," which closely connect to recent theories about learning and the human brain. She explained how the purpose of education is a lifelong acquisition of wisdom, and that prior learning is essential to future learning. Learning is a dynamic process of constructing meaning through personal inquiry and pattern formulation. Students must be fully engaged through the reflective exploration of essential and deeply human questions.

In his constructivist dimensions of learning model, Robert Marzano (1992) described how acquiring and integrating knowledge occur when information already known and understood is used to make sense out of newly acquired knowledge. Then knowledge is extended and refined as students develop new insights, ideas, and discoveries about things they already know through questioning, comparing, classifying, inducing, deducing, and analyzing errors. Finally, students learn to use new knowledge meaningfully as they reflect on the knowledge and apply it to their everyday lives. Using knowledge meaningfully involves the use of higher-level thinking skills, decision making, investigation, experimental inquiry, problem solving, and invention.

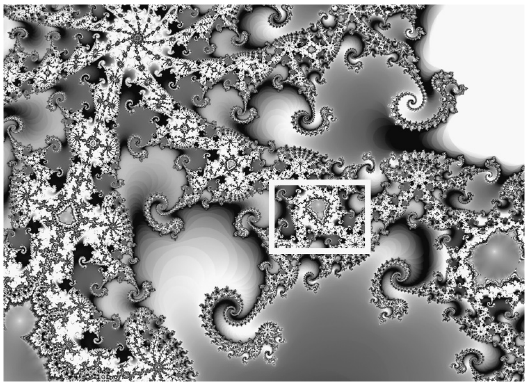
For example, in a third- or fourth-grade lab about static electricity, begin with a short discussion about personal experiences the students have had with this form of electrical energy. Many students have at some time rubbed balloons against their hair and experienced it standing on end. Students usually mention being shocked while walking across a rug, touching a light switch, or when combing their hair on a dry day. Groups of four students can go on to experiment with a variety of materials to investigate static electricity as an energy source. They can try various methods to get balloons to stick to the wall. They can rub two balloons with pieces of wool



Fractal



Magnification of Fractal Inset at Left



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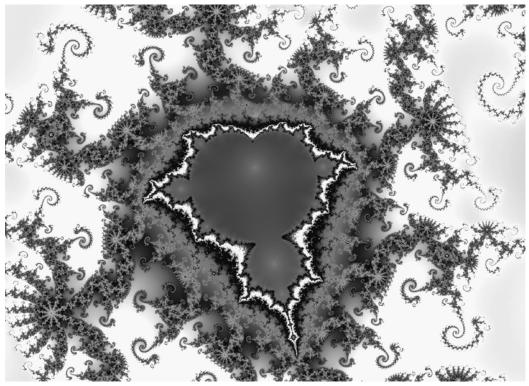

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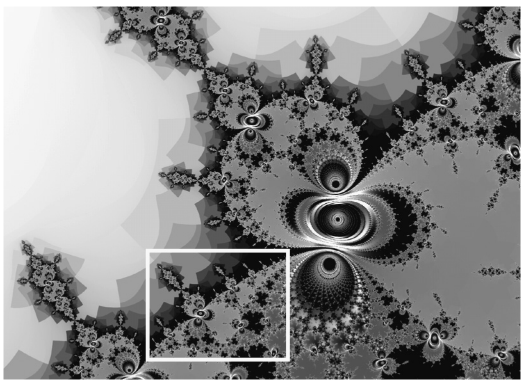

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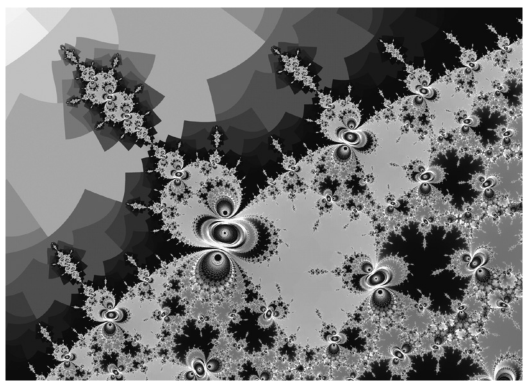
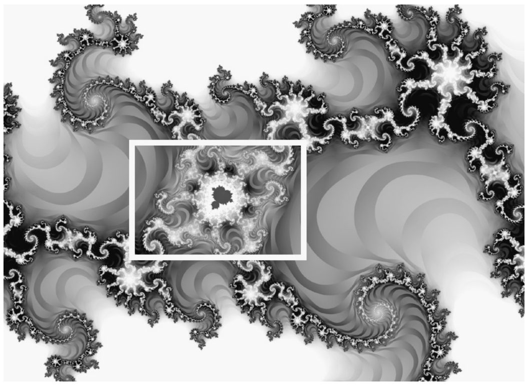


Magnification of Fractal Inset at Left

Figure 1.3

Fractal images by Jock Cooper. © 2006. Used by permission.



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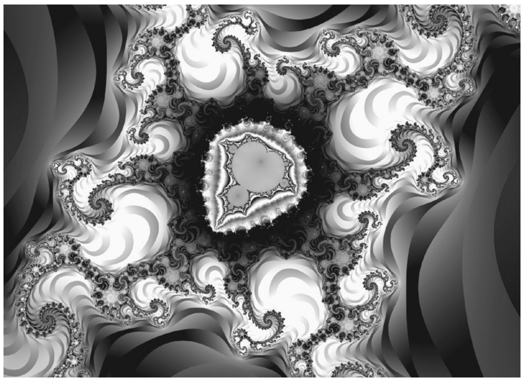

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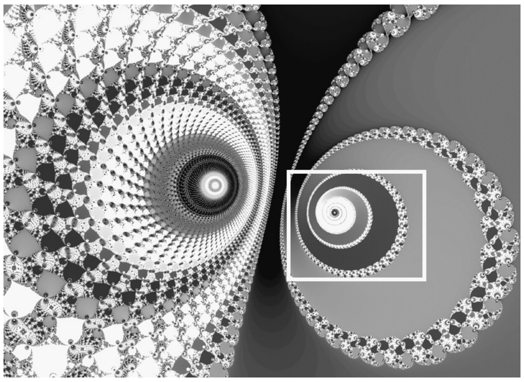

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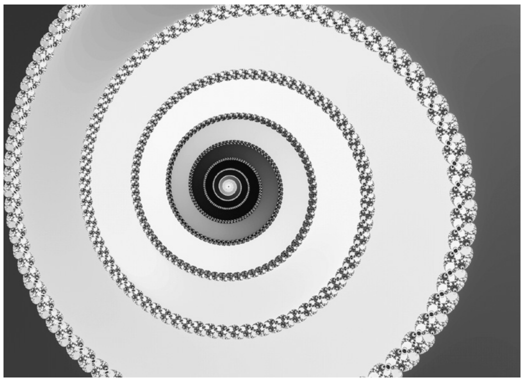
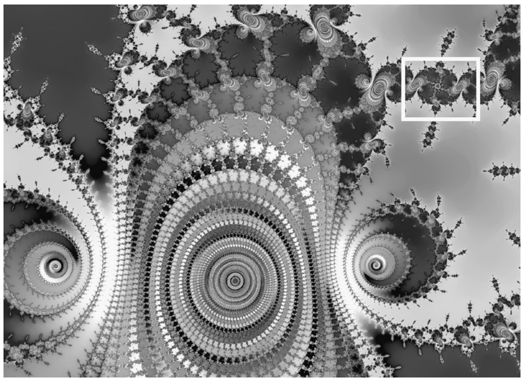
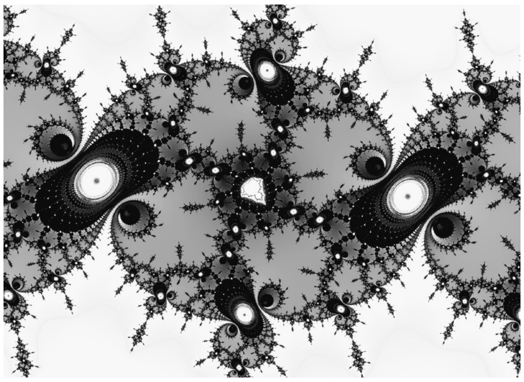


Magnification of Fractal Inset at Left



Fractal



Magnification of Fractal Inset at Left

Figure 1.3

(Continued)

Fractal images by Jock Cooper. © 2006. Used by permission.

or fur and watch the balloons push away from each other. Show the students how to charge a comb with static electricity by rubbing it with wool or fur. Then bring the comb close to little bits of tissue paper. Invite students to create a static electricity experiment of their own using confetti, puffed rice, salt, pepper, dried parsley flakes, saran wrap, tinsel, and the like. Throughout the lesson encourage communication and positive interaction.

After cleaning up the materials, ask the students to explain their exciting discoveries, sharing what they have learned about static electricity. Reflect upon their experiences by writing and/or drawing pictures of their new understandings. What additional activities would the students like to try? Perhaps give them ideas to do and share with family at home.

Continue the lesson the next day with a discussion about lightning. What is lightning? What causes it to occur? What are some lightning safety rules? How do the students feel during a bad thunderstorm? Darken the room and rub two balloons against some fur. Then hold the balloons, almost touching, and observe a spark jump between them. When clouds and objects on the ground become charged with static electricity, electrons jump from an area with a negative charge to a positively charged area, producing a lightning bolt!

Howard Gardner (1991) concluded that children often grow into adults harboring deeply entrenched concepts about the world. For example, they might believe that lightning and thunder occur when two clouds bump into each other or that the Sun travels across the sky each day and disappears at night. Gardner contended that education for understanding occurs only when students integrate prescholastic with scholastic and disciplinary ways of knowing.

> All understandings are partial and subject to change; far more important than arrival at a "correct view" is an understanding of the processes whereby misconceptions are reformulated or stereotypes dissolved. (Gardner, 1991, p. 152)

Even with advanced training, students too often regress back to their view of the world from the perspective of a 5-year-old child (Gardner, 1991).

When students leave school at the end of a school day, teachers should not assume that students make the connection between the taught lessons and their lives outside of school. Cynthia Crockett (2004) discussed how educators must lead students to deeper scientific understanding by helping them first to identify their ideas and then to reflect on evidence to support their ideas.

> Understanding science requires students to learn a vast array of facts, processes, and skills. To ensure that this understanding is accurate, we need to know what the students think and why they think it. . . . As educators, we can expose students' misconceptions and help them unravel their false ideas. We must then provide students with the opportunity and support to develop a more accurate set of beliefs on which to build true scientific understandings. (Crockett, 2004, p. 37)

Student understanding is constructed through thoughtful discussion and careful assessment.

Many misconceptions appear when students are asked to explain how the Earth, Moon, Sun, and stars relate to each other. To raise their level of understanding, students begin by comparing four ancient models of the Earth, explaining how the Earth, Moon, Sun, and stars relate and interact. They invent their own ancient models of the world to describe the motion of the Sun in the sky, how it rises in the East, goes overhead, and then sets in the West. Next, they present their ideas and, after each presentation, encourage their classmates to ask questions and to engage in further dialogue. Students see that many different models can be used to explain the same set of observations. Later, students work in cooperative groups to discuss the implications of the ball-shaped Earth model to help them develop insights about Earth's shape and gravity. They support their ideas with arguments or demonstrate using globes (Sneider, 1989). Finally, students can participate in kinesthetic activities to physically act out the relationships of the Earth, Moon, Sun, and stars. Through the process of reflection, and by confronting their misconceptions, students reconstruct their understanding of Earth and beyond, encompassing a more sophisticated and scientific view.

Many perspectives and dimensions of learning exist. Science educators must encourage students to look at the world from differing points of view, nudging them toward complex, real-life experiences and creative problem solving. Learning takes on a fractal scaling dimension as newly constructed meanings intertwine, web, and build—the old contained within the new like nesting boxes.

> One has to look for different ways. One has to look for scaling structures—how do big details relate to little details. . . . Somehow the wondrous promise of the Earth is that there are things beautiful in it, things wondrous and alluring, and by virtue of your trade you want to understand them. (Feigenbaum in Gleick, 1987, pp. 186–187)

Getting the "right answer" is not as important as the critical-thinking skills that students develop while struggling to apply their mental models to real and imaginary situations.

Search for Repeating Patterns and Different Perspectives

The world is filled with many natural patterns, the result of nature's efficiency. Nature uses the same pattern over and over again, if the pattern is suitable for the purpose that nature has in mind. The spiral is one of the many basic designs that seems to repeat itself in many natural systems. Spiral patterns on hurricane maps, spiral-shaped galaxies, spirals on pinecones, and spiral-shaped seashells and snails are a few examples. Additional natural patterns include branching, ripples and waves, hexagons in beehives, bubbles, and radial symmetry.

Several of Caine and Caine's (2006) brain-based learning principles deal with the human brain's strong need to make sense of the world through patterning (see Introduction). The Caines believe that through patterning the search for meaning happens naturally. Making sense of life experiences is innate as the brain works to understand patterns and express them in uniquely creative ways. Patterning and interconnecting are central to learning and understanding. Human brains want learning to occur, making connections and seeking out patterns from the constant information flow of life experiences. A combination of novelty and familiarity are needed in a learning environment to ensure that true understanding takes place. Embedded activities linked to students' prior experiences lead to meaningful and genuine learning situations.

In their work to initiate a paradigm shift in education, Caine and Caine (1997a) identified three perceptual orientations, each reflecting a different perspective for teaching and learning. Perceptual Orientation 1 is the traditional approach, with the teacher in control and a fragmented curriculum. Perceptual Orientation 2, a more complex perspective, uses complex materials and engaging experiences to create meaning. With this approach, teachers may see links between some subjects while maintaining strong conceptual boundaries between others. According to the Caines, the ideal perspective is Perceptual Orientation 3, which they envisioned as brain based, learner centered, and more dynamic, "with educational experiences that approach the complexity of real life" (p. 25).

> Teachers at this orientation are open to the multiple possibilities of interconnectedness. They know or sense that every subject in the curriculum is a way of organizing human experience and is therefore interconnected at a deep level. Thus, they do not see the ideas in the curriculum as standing alone—they relate them to life experience. . . . (Caine & Caine, 1997b, pp. 128–129)

Caine and Caine averred that the key to transforming education lies in the ability of teachers to transform themselves as they search for and evaluate the many options open to them. The Caines (1997a) identified five indicators of the instructional approaches that seem significant to differentiate among the three orientations; instructional objectives, use of time, sources for curriculum and instruction, dealing with discipline, and approaches to assessment. No one of the five indicators in isolation, however, can determine a teacher's educational approach. As teachers grow into Perceptual Orientation 3, instructional objectives become more connected to life experiences; assessment, more authentic; and use of time, more flexible and student centered. These teachers use multiple sources for curriculum and instruction and replace traditional discipline procedures with the creation of collaborative communities focused on order, trust, and respect.

Each day teachers unravel their way through endless lists, schedules, meetings, lesson plans, and science curricula, which can easily lock them into a structure that is too confining for the nonlinear, constantly evolving world. Teachers must see what is going on at every level for the clarity of vision to make change possible. They need to step back from their science curricula and instructional strategies to gain a new and perhaps enlightening perspective and to look for themes and patterns instead of isolated causes and facts.

APPLICATION FOR BRAIN-COMPATIBLE SCIENCE

The following lesson plan, *Changing Perspectives,* introduces the chaos theory principle of fractals into a science/geometry lesson involving a simple snowflake shape viewed from several different perspectives. Teachers discover that changing the scale redefines the snowflake's outer edge much in the same way that the human brain is shaped by life experiences. Following the lesson plan is a web to incite more creative ideas (Figure 1.10) and a chart to navigate the road to change in the brain-compatible, science classroom (Figure 1.11).

Lesson: Changing Perspectives

Chaos Theory Principle: Fractals

Grade Level: K–8, Gifted and Talented

Chaos Connection

- The Koch snowflake is a fractal pattern that repeats itself on increasingly smaller scales.
- The perimeter of the snowflake is infinite even though the snowflake always fits within a finite area.
- When viewed from a variety of perspectives, coastlines become increasingly longer as more detail is included in the measure.

Curriculum Connection

- Geology
- Weather
- Measurement
- Maps
- Geometry

National Science Education Standards

- Content Standard A; 1–9
- Content Standard D; 1, 2

Objectives (Students will . . .)

- Investigate the Koch snowflake as having an infinitely long perimeter and compare it to a lake or coastal shoreline viewed from a tiny insect's perspective.
- Use critical thinking skills to analyze the potential complexity of a simple fractal shape.

Materials

- ❑ Koch snowflake transparency (use Figure 1.4)
- ❑ Koch snowflake handouts (Figures 1.5, 1.6, 1.7, 1.8, and 1.9)
- ❑ Thin string or yarn
- ❑ Compasses
- ❑ Transparency circles and/or laminated construction paper circles
- ❑ Crayons or markers
- ❑ Scissors

Preactivity Discussion

1. Introduce the Koch snowflake with the transparency, triangle, and six-pointed star, followed with the more complex snowflake figures (see Figure 1.4). Ask the students to look closely at the outside edges of the snowflakes and to explain how they are drawn. What repeating pattern do they notice? (Students will see triangles and will comment on their size and the number).

Koch Snowflake Views

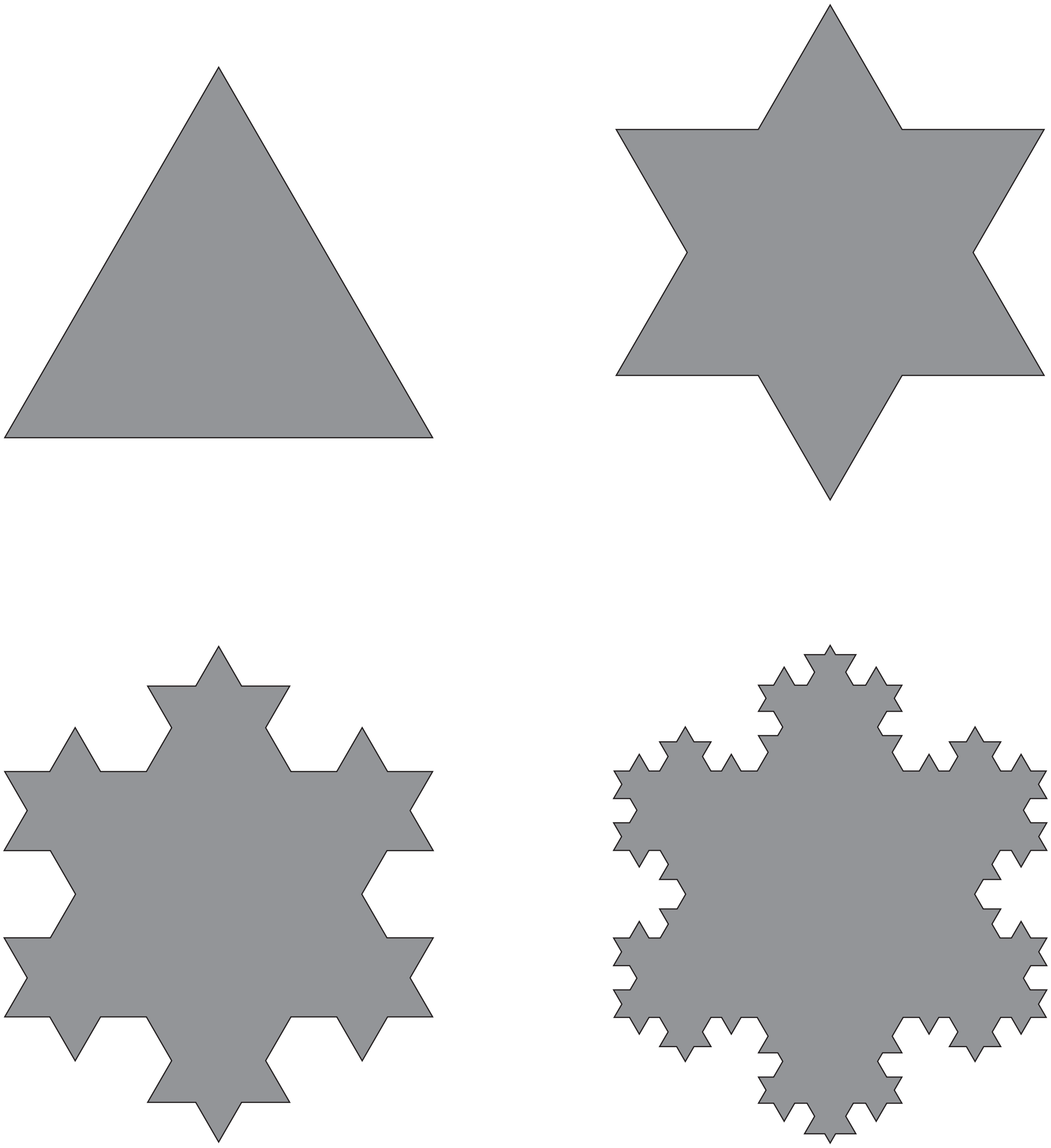


Figure 1.4

2. Discuss the following questions with the students: If we lay a string around the large triangle and then around the six-pointed star, which string will be longer? Why? How will the length of the string change if we lay it around the other two snowflake figures? Predict what will happen when we draw circles around the triangle, star, and snowflake figures. How will the size of the circles change?

Procedure

Organize the students into cooperative groups of two to four. Pass out handouts of the circle, the triangle, the star, and the two snowflake shapes (use Figures 1.5, 1.6, 1.7, 1.8, and 1.9). Invite the students to experiment freely with the patterns. Give them the following ideas to get started:

1. Color and cut out the snowflake patterns.
2. Experiment with positioning the patterns in different ways.
3. Hold the patterns up to the light to discover how they fit together.
4. Construct more triangles within triangles.
5. Use string to measure around the edges of the figures to discover which shape is the longest.
6. Use a compass to draw circles around the triangle and snowflake shapes.
7. Use transparency circles or laminated construction paper circles to discover that the figures all fit inside a finite area.

Closure

1. Circulate among the groups and discuss the students' observations. Ask students why the patterns always remain within a finite area of congruent circles.
2. Use the transparencies on an overhead projector to illustrate the relationships among the circle, triangle, six-pointed star, and the two Koch snowflake views.
3. Encourage students to share their pattern experiments.

Questions and Extensions

1. Ask students to think about how many self-similar snowflake figures they could make. How small will the sides eventually get? Can we keep on going to the molecular level? To the level of atoms? To infinity? Just how long could the outside of the Koch snowflake get? (Infinitely long!) What special tools might we eventually need for measuring?
2. Continue with a discussion of a trip to the beach. How long is the beach shoreline? How would the shoreline look when viewed from the air? From up on a bluff looking down? From a student's point of view? From a snail's point-of-view? Which view is the longest? What if a small sand insect travels around each tiny pebble and grain of sand? If we could follow the shoreline journey of a microscopic organism, how long would that take? Will the edge of the snowflake ever be a perfectly straight line? Why not?

3. Discuss why various encyclopedias, atlases, and other reference books may give different mileage for the same shoreline or boundary. How many fine details do maps tend to smooth out? What kinds of maps give the least detail? The greatest detail?

4. Have students choose one of the Koch snowflake patterns (Figures 1.8 or 1.9) and complete the following activity. Cut carefully around the outside edge. Fold first in half and then in sixths, matching the pattern on the edge. Cut lacy patterns from the folds. Open and compare with classmates to see that no two snowflakes are ever the same.

5. Repeat the triangle exercise with a square. Have students draw a large square first and two more complex views. All three must fit within congruent circles.

6. For a bodily/kinesthetic, experience or for younger students, enlarge the patterns so that they are big enough for the students to walk around. Time how long it takes to walk once around each. Provide rhythm by playing music in the background to keep the students' steps consistent.

7. Have students compare a map of the same area drawn on three different scales. Which map shows the greatest detail?

Technology Connection

Browse the following Web sites to find exciting new ideas for teaching science. Remember to use links and keywords to search even more science-related sites.

A Fractals Lesson
http://math.rice.edu/~lanius/frac/

The Spanky Fractal Database
http://spanky.triumf.ca/

Fractals (many images)
http://astronomy.swin.edu.au/~pbourke/fractals/

Exploring Fractals
http://www.math.umass.edu/~mconnors/fractal/fractal.html

These Web site addresses were accurate at the time of printing; however, as these sites are updated, some of the addresses may change.

Circle

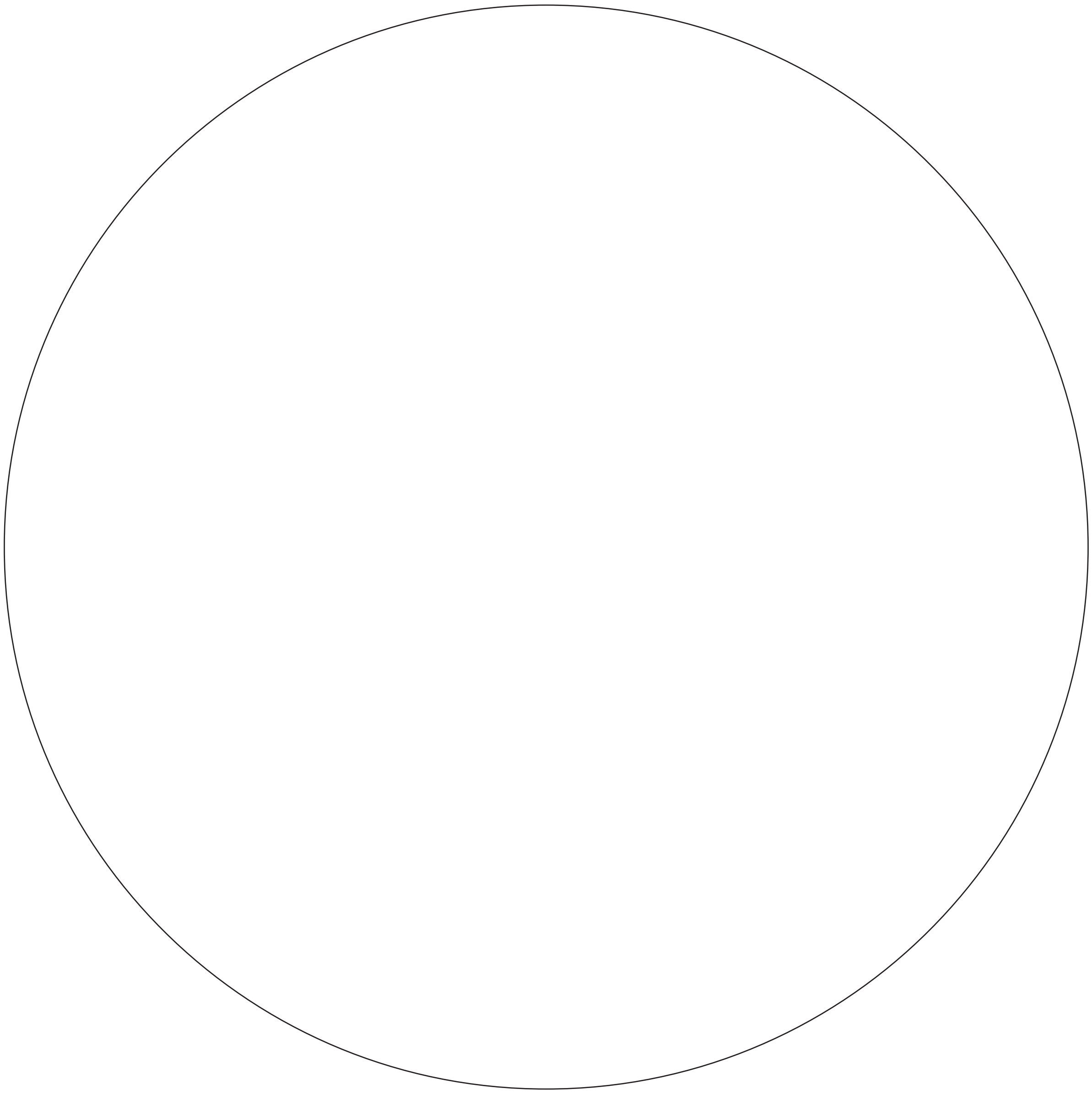


Figure 1.5

Triangle

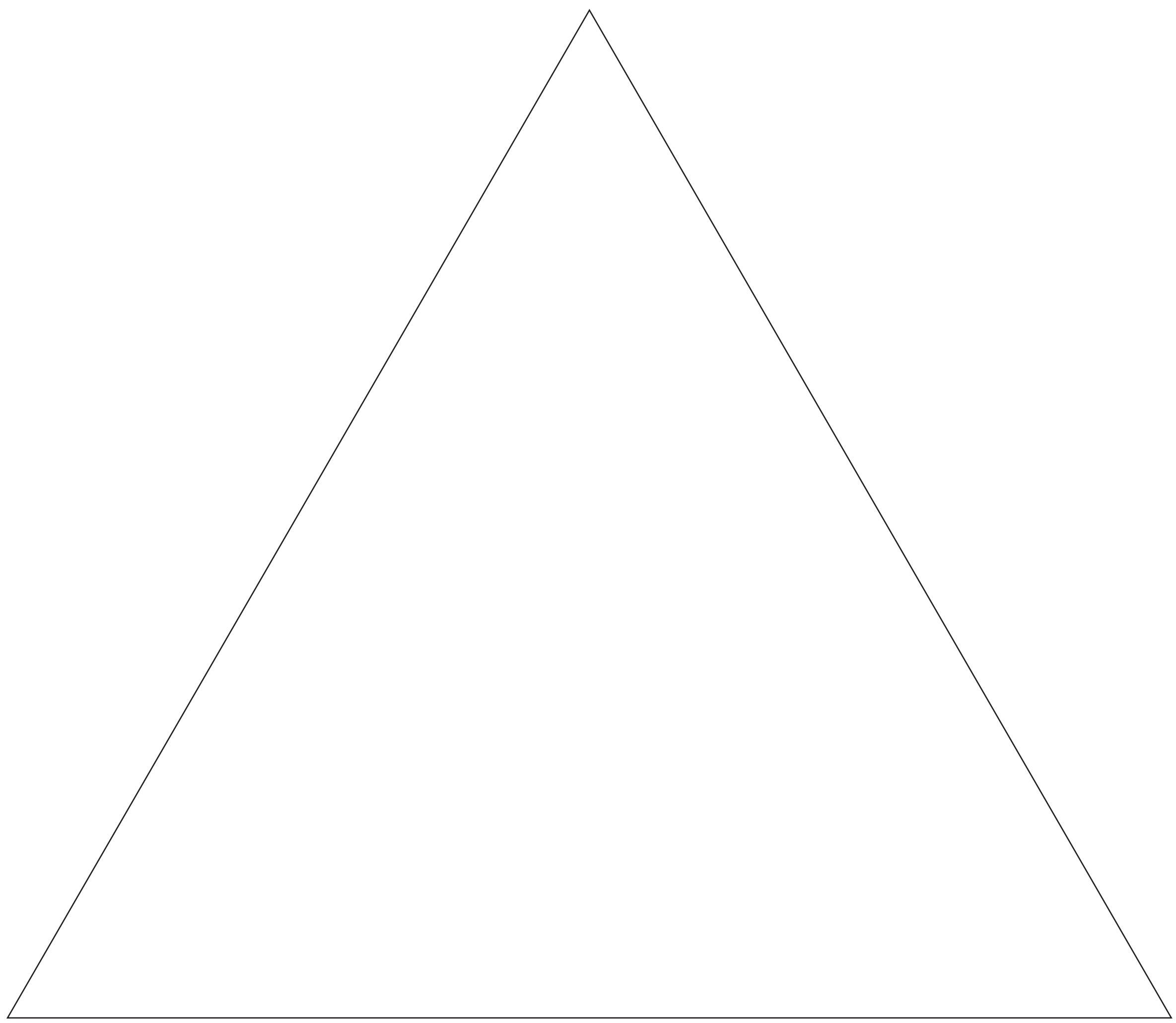


Figure 1.6

6-Pointed Star

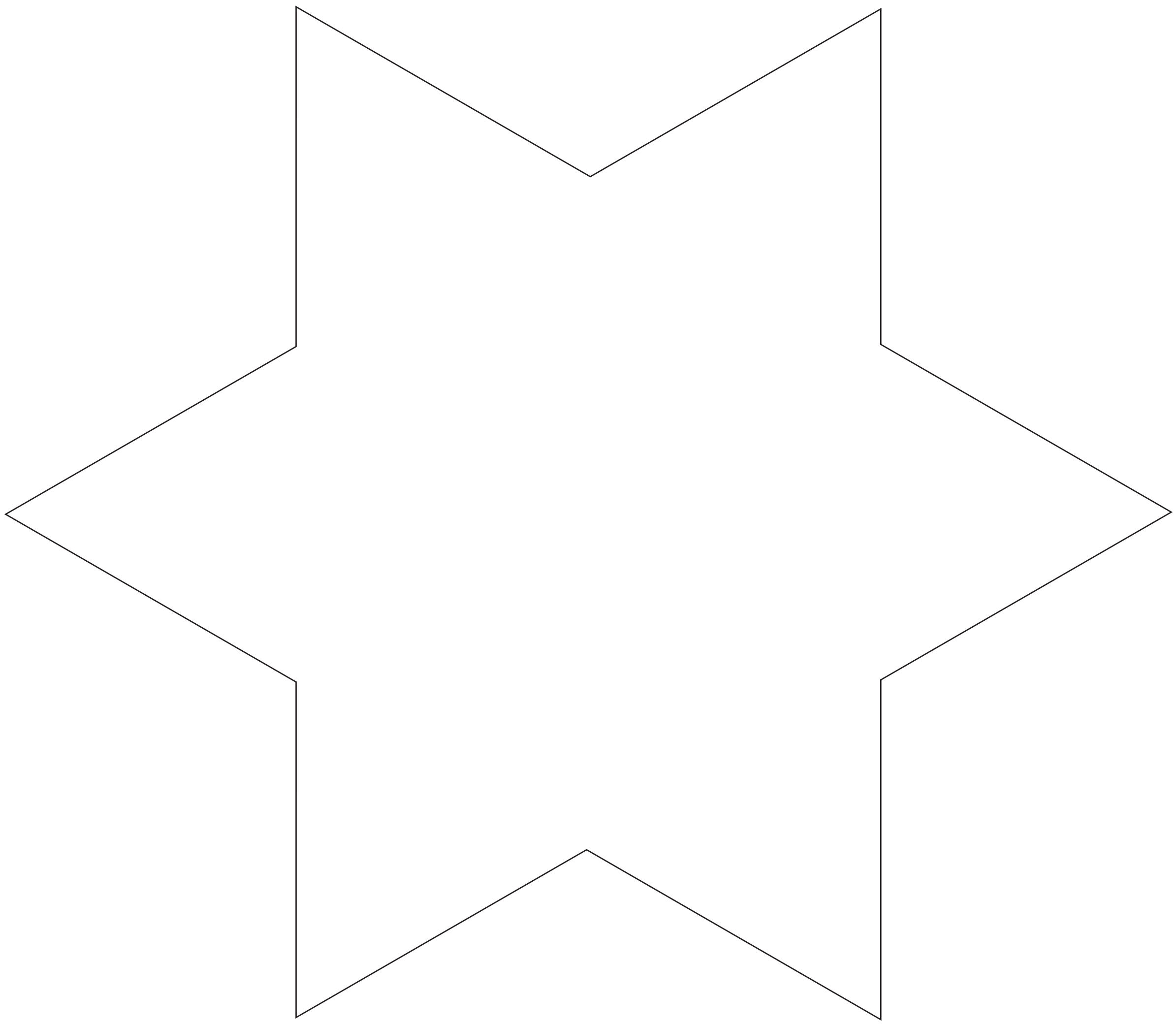


Figure 1.7

Koch Snowflake 1

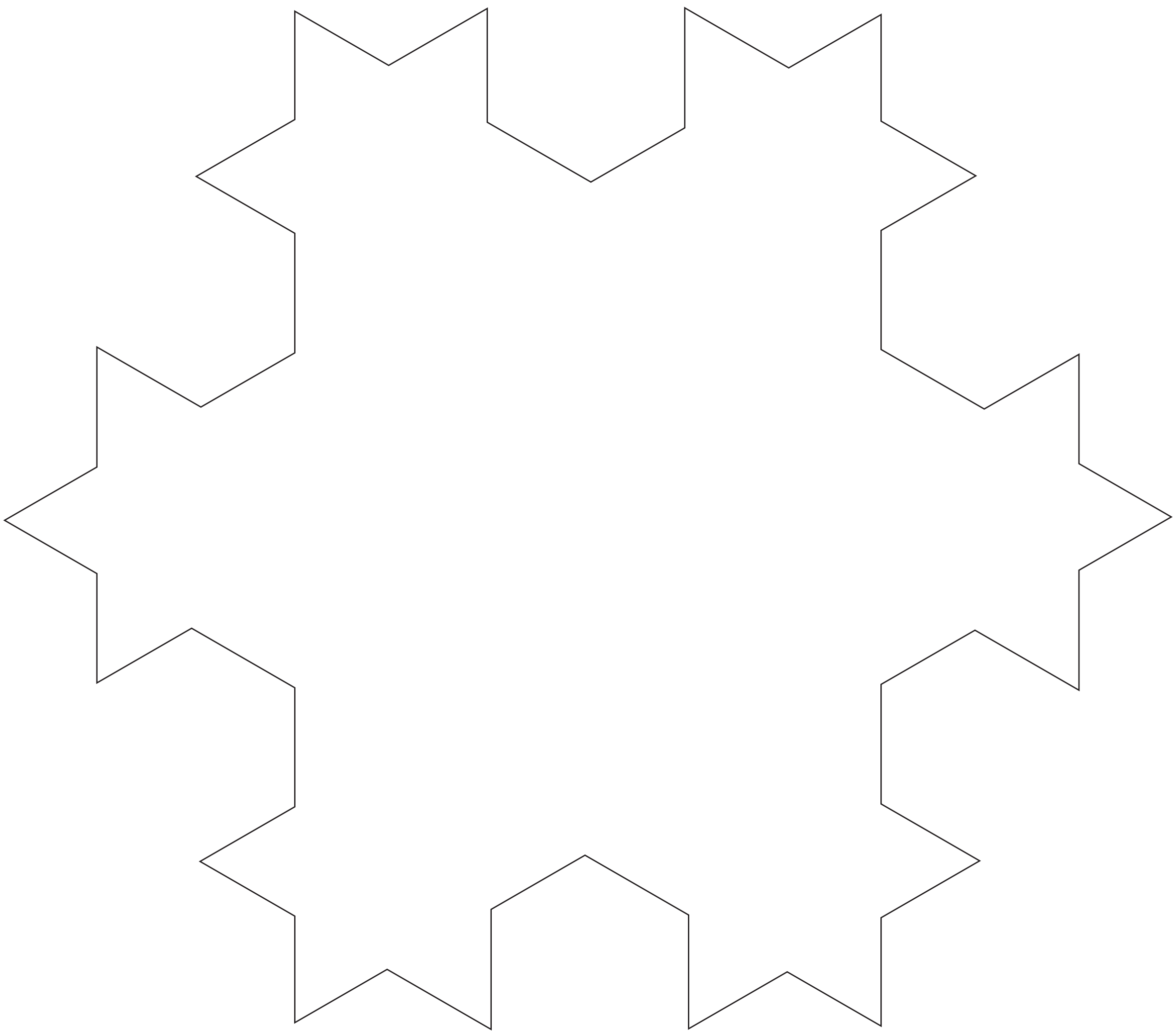


Figure 1.8

Koch Snowflake 2

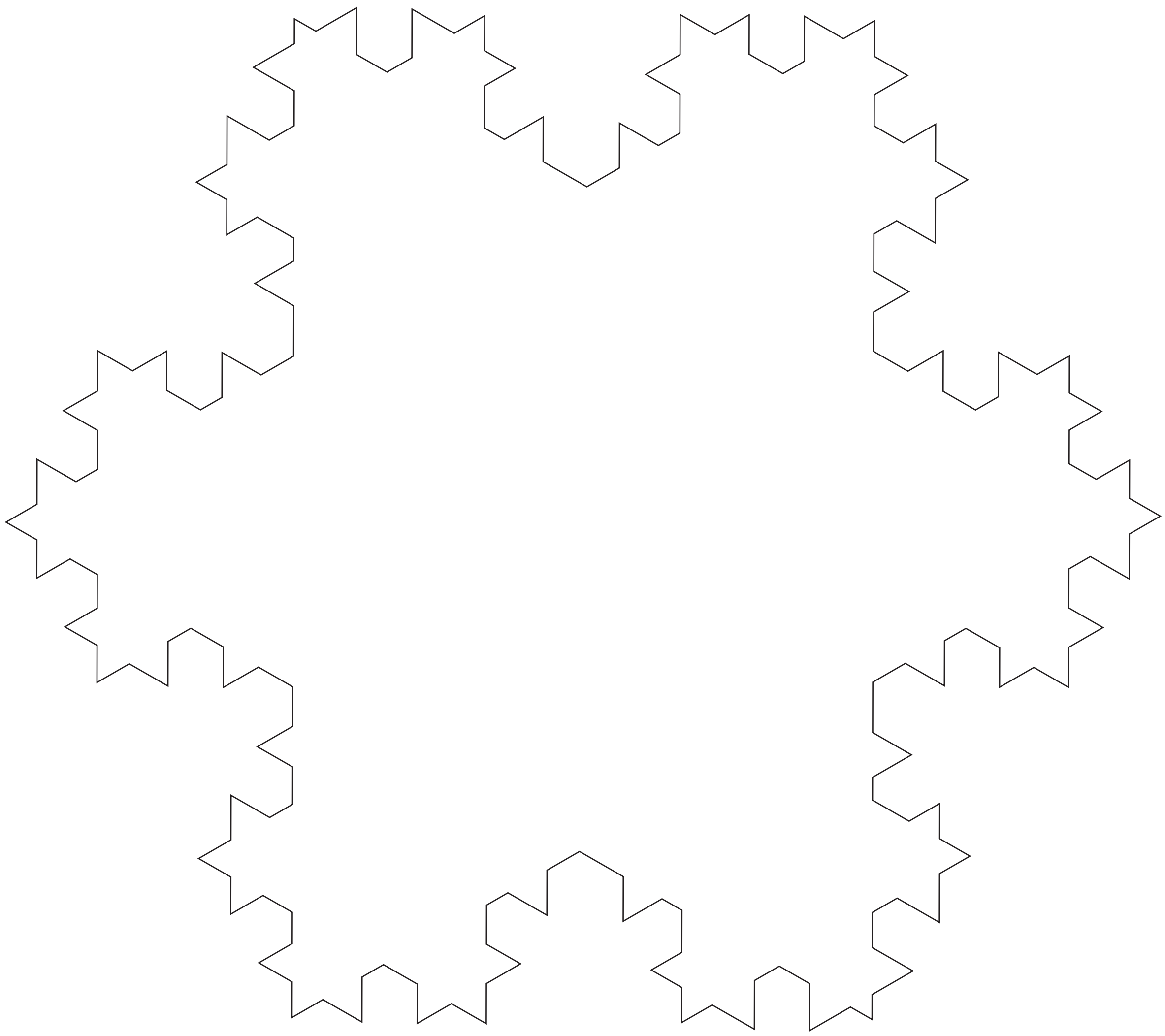


Figure 1.9

Web for Applying Fractals

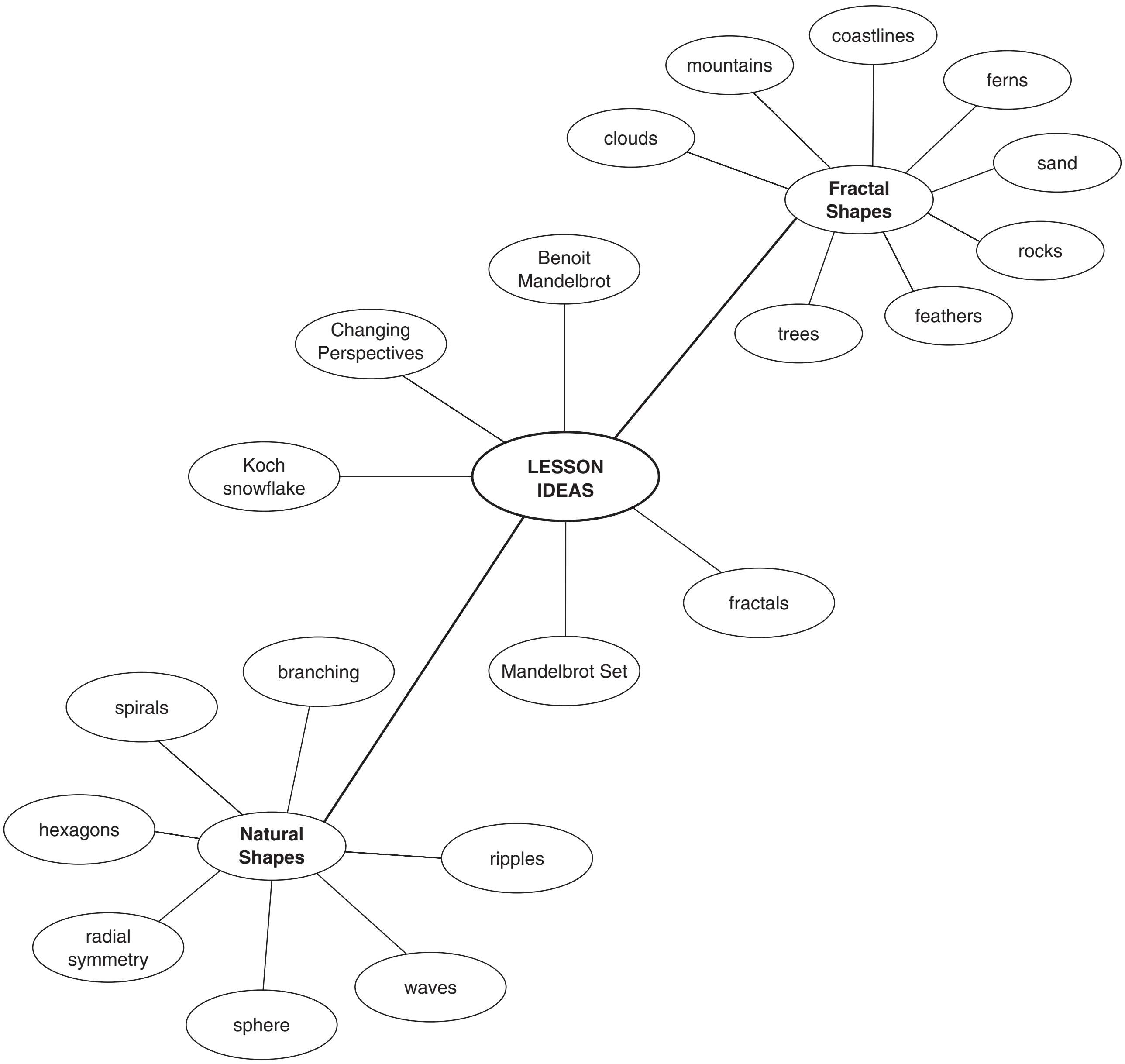


Figure 1.10

NAVIGATING THE ROAD TO CHANGE IN SCIENCE EDUCATION

Fractals

Too Much Order	On the Edge	Too Much Chaos
• Overplan, structure, and assess.	• Simplify by working with life's natural tendency to organize.	• Do not bother to structure, organize, or plan.
• Focus on isolated causes and facts.	• Look for meaning in patterns and themes.	• Overemphasize the importance of patterns and themes.
• Overemphasize previous learning with tedious review.	• Construct new meaning from old with higher-level thinking skills.	• Ignore previous learning.
• Disregard student misconceptions.	• Confront student misconceptions to raise level of understanding.	• Allow discussion of student misconceptions to shift away from the focus of the lesson.
• Dwell too long on connections.	• Connect instructional objectives to life experiences.	• Forget to make connections.
• View science education with tunnel vision.	• View science education from a variety of perspectives for a balanced outlook.	• View science education from an idealistic view by going with every new trend.

Figure 1.11