

# 1

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## A Glimpse at Mathematics Instruction

In this chapter we will discuss the following:

- ❖ An Algebra Classroom Vignette
- ❖ A Geometry Classroom Vignette
- ❖ Best Practices for Teaching Mathematics

The teacher walks into class and sees 28 faces staring back at him, all sitting in rows, one behind another. The room is barren except for the chalkboards on three sides and a few books on the teacher's desk. Today, like every other day, the teacher asks if there are questions from the previous night's homework. If there are questions about a particular problem, he works through the problem on the chalkboard and moves on to the next one. Following this daily routine, the teacher takes out his book, turns to the next section in the text, and lectures as he writes notes and works examples on the board. Rarely does he turn around and even look at the students, much less ask them if they understand his lecture. Finally, after he has completed three examples on the chalkboard, he writes the following assignment on the board: "problems 1–45 on p. 17; quiz on sections 6.1–6.5 on Friday." He then sits at his desk as the students work quietly on their homework until the bell rings to release them from class.

Does this scenario seem familiar to you? How did you feel learning math this way when you were a student? Whether this has been your personal experience or not, you probably realize that there is more to teaching mathematics than what we have just described. New research and technology have brought into question many of the methods of the past, and progressive classrooms of today are quite different from the classroom described above. This book can help you prepare for the many aspects of teaching mathematics so that you will be able to make informed decisions on what to teach as well as how to teach it. We begin by taking a look at two classes with teachers well versed in the latest methodologies of teaching mathematics.

## TWO ILLUSTRATIONS

The first class is having an Algebra I lesson on slope and proportional reasoning; the other a seventh grade lesson on surface area. Both teachers are experienced and demonstrate many exemplary practices in the teaching of mathematics. As you read, see if you can identify these teaching practices.

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### VIGNETTE 1: ALGEBRA I—SHAKE ACROSS AMERICA

*It is 10:12 on a Thursday morning, and Mrs. Malloy is busy gathering materials for her 10:15 Algebra I class. As the students enter the room, they are randomly assigned to sit at one of the eight tables. Four students sit at each table, and each table is provided with the following materials: a tape measure, a stopwatch, and four copies of the assignment shown in Figure 1.1.*

*This activity involves a real-world problem-solving scenario in which students are asked to determine how long it will take for a handshake to travel from the east coast to the west coast of the United States. Students begin by collecting data on how long it takes a small group of students to shake hands. They then use both proportional and algebraic reasoning to estimate how long it would take to complete the handshake. This activity incorporates meaningful mathematics as well as several of the big ideas in middle school and high school mathematics. It was selected because it involves active data collection, a variety of approaches, and a range of acceptable answers. In fact, there is no one “real” answer, since such an answer would be different every time the event occurred. The most important part about this activity is the mathematical reasoning and methods used to get an answer, not the final answer itself. See Appendix A for sample data and solutions.*

**Figure 1.1** Shake Across America

This project should be completed in groups of three to four students. On a separate sheet of paper, please respond completely and clearly to the questions listed below.

**Shake Across America!**

Materials: tape measure, stopwatch

An environmental group is organizing a “handshake across America.” The group plans to have people line up from New York to Los Angeles and pass a handshake from east to west.

**Data Collection**

1. Have several members of your group form a line. Count how many students are in the line and measure its length.
2. Start a handshake at one end of the line. Use a stopwatch to find out how long the handshake takes to reach the end of the line.
3. Repeat steps 1 and 2 for four different lengths of the line (e.g., 3, 6, 9, and 12 students). You will need to get together with another group or two.

**Proportional Reasoning**

4. Use your results to estimate the number of students per foot.
5. The road distance from Los Angeles to New York is 2,825 miles, or almost 15,000,000 feet. About how many people need to be in the line?
6. Now estimate both the speed of the handshake (feet per second) and the ratio of seconds per foot.
7. How long will it take to have the handshake go across America? Explain how you found your answer.
8. Explain how you can use proportions to solve this problem.

**Algebraic Reasoning**

9. Produce a scatter plot of the data you collected, with length as your independent variable and time as your dependent variable.
10. Determine the line that best fits your data.
11. How does the slope of your line relate to what you found in the proportional reasoning section?
12. In theory, what should the y-intercept be? Why?
13. Use your line of best fit to get an estimate of how long the handshake across America will take.

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*Source:* Adapted from Charles, Dossey, Leinwand, Seeley, & Vonder Embse, 1998, p. 344.

*Mrs. Malloy introduces the lesson by showing a YouTube video of a Hands Across America infomercial from the 1980s. There is a short discussion on the economic situation of the 1980s that prompted the Hands Across America event. (Amanda McKee, personal communication, December, 2008). Mrs. Malloy then directs the students toward the handout that has been passed out.*

*Students recognize this as a group activity, and questions originally center on the roles of each group member. Since they have done cooperative learning activities before and are familiar with the different roles that members of each group need to play, Mrs. Malloy allows her students to brainstorm as a class to come up with the roles of the group. The students come up with following roles: timer (individual who keeps time with the stopwatch), recorder (person who records the results), and reporter (student who reports the results).*

*Mrs. Malloy has anticipated other questions, so she has the students demonstrate as a class how to collect data for the activity. She has twelve students line up, and another student times the handshakes as three, six, nine, and twelve students shake hands. A recorder records the times as the handshakes are being completed. As the rest of the class observes this demonstration, they also begin to see a need for a “clapper,” someone who would clap to keep the handshakes going at a consistent rate. They then begin the activity.*

*As the students begin working in their groups, some raise their hands. Mrs. Malloy walks around the room and hears similar versions of the same questions. After addressing the questions with one group and hearing the same questions from other groups, she decides to bring the class back together and have a class discussion about the questions “How far apart should we stand?” and “How fast should we shake our hands?” After several groups are given a chance to explain their viewpoints on the matters, the class concludes that as long as they are consistent within their own groups, it is not critical that each group make the same decisions.*

*The students proceed with the activity and complete the data collection section without any other questions. As they move into the proportional reasoning section of the activity, they once again have questions. In this section, students are asked to estimate the number of students per foot, the speed of the handshake (feet per second), and the ratio of seconds per foot. The two main questions that arise focus, first, on how to deal with the many ratios that can be created, and, second, what to do with the data for three, six, nine, and twelve students. Mrs. Malloy has intended for students to use the data for twelve students when solving proportions; she has asked them to collect data for three, six, and nine students so that they have data for the scatter plot and line of best fit in the algebraic reasoning section. Yet she does not tell them this directly, since she wants to encourage her students to think about how the data can be best used. As she circulates from group to group, she listens to students explain their reasoning and guides them with questions of her own if she realizes that they are off track.*

*In the final section of the activity, students are asked to use data to produce a scatter plot and line of best fit in order to predict how long it could take to complete the handshake across America. Since the use of technology is encouraged in Mrs. Malloy's class to develop understanding, students feel comfortable using it as they see fit. While some students choose to create their scatter plots using graph paper, others choose to use graphing calculators. As Mrs. Malloy walks around the room, she answers questions from the groups and comments on their work. She also notes that many of the groups are coming up with a variety of answers. At the end of class, she tells the students that they will continue this activity the next day and that groups will be asked to report their findings verbally to the class and prepare a write-up to turn in to her.*

*The next day, students spend the first half of the class period finishing up the activity and creating posters to present their results. Mrs. Malloy checks the progress of each group and works with individual groups who still have questions. Students are then chosen randomly by a roll of a die to determine the order in which they will make presentations. Mrs. Malloy asks a member of each group to present the group's poster. They are also allowed to bring another student from their group if they want assistance.*

*As the groups report their findings, it becomes very clear there is a wide range of answers from the class. After the second group presents, the students begin a discussion about how there could be such a difference in the "right" answers. Mrs. Malloy has carefully chosen this activity because of its open-ended problem-solving nature; she wants her students to recognize the importance of justifying their work and to realize that, in this problem, it is in justifying each step of the process that the answer is determined to be correct or incorrect. Much of the remaining class time focuses on the process through which each group made its findings. Some time at the end is reserved to discuss the similarities and differences of the proportional and algebraic reasoning approaches as well as to make explicit the connection between the ratio of seconds per foot in the proportional reasoning section and the slope of the line in the algebraic reasoning section.*

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## DISCUSSION

In this section we will focus on the following points:

- ❖ Meaningful Mathematics
- ❖ Making Connections
- ❖ Teacher as Facilitator
- ❖ Communication
- ❖ Positive Classroom Culture
- ❖ Effective Instructional Strategies

Now that you have read the vignette, we want to steer you back to thinking about the exemplary practices employed in this lesson. First of all, we review this activity through the lens of meaningful mathematics. Shake Across America addressed many areas of important mathematics and encouraged students to make connections to other areas of mathematics. These considerations are very important to Mrs. Malloy, since she works hard to teach all of the standards required by her district and state, and she often feels pressed for time to accomplish everything. With activities such as Shake Across America, she is able to pull together several concepts in a single lesson. You may have noticed that students investigated the same problem scenario using both proportional reasoning and algebraic reasoning, which helps to make explicit the connection between the ratio of seconds per foot and the slope of the regression line. Numerical and graphical representations were used, as were symbolic representations when finding the line of best fit. Students analyzed the data collected and used their regression line to help them predict how long the handshake will take to complete. Using all of these concepts in a single lesson helped students to construct links between mathematical understandings.

Beyond the specific mathematical concepts and connections, there were several teaching practices employed that are worthy of note. We continue the discussion of this activity by addressing these practices next.

You may also have noticed the stark contrast between the role of lecture in Mrs. Malloy's classroom and in the classroom scenario at the beginning of this chapter. Mrs. Malloy did not spend the majority of class time lecturing to her class. In fact, she was much more of a facilitator than a lecturer. Although Mrs. Malloy does lecture at times, she is conscious about the time she spends at the board in front of the class. She wants to make sure that students are given plenty of time to explore new material, to create their own understandings, and to connect new understandings to existing knowledge.

The facilitation of understanding played out throughout the activity as Mrs. Malloy guided her students. In the vignette you observed students asking questions about the assignment, yet Mrs. Malloy did not directly answer the questions. Instead, she gave her students opportunities to think about their questions, come up with responses, and defend their thinking. Mrs. Malloy believes exploration to be a valuable experience, and, in her opinion, lecture does not afford students enough opportunities to develop their mathematical reasoning skills.

Cooperative learning and communication are important components that accompany the philosophy of teacher as facilitator. After all, if the teacher is not going to give the answers, how are the students going to

develop understanding? While Mrs. Malloy did not simply dispense all her knowledge to her students, she allowed for plenty of discussion, both in small groups and the whole class, so that students could help each other to make sense of the mathematics in the activity. In more than one instance, students arrived at conclusions without her direct instruction, and student discourse had actually facilitated the conclusions.

Mrs. Malloy was also consciously developing a positive classroom culture, conducive to enabling all students to learn important mathematics. She did this in many ways. First of all, she selected an activity that students could relate to. It was her hope that her students would be engaged in Shake Across America, would enjoy learning mathematics because of this activity, and would feel better about themselves as learners as a result.

Next, Mrs. Malloy used many strategies to include all learners and to promote equity in her classroom. She grouped her students; she does this so that everyone can experience working with different partners throughout the year. Students were also given different roles in the group so that all could participate and to lessen the likelihood of one person dominating the group. In addition, when students explained their group's work, they were allowed to bring a partner. This was done so that any student who might feel uncomfortable for any reason would have another student for support. Often, students who struggle with their English speaking skills or do not have confidence in their mathematical abilities will shy away from communicating with the large group and thus miss out on the important experience of communicating their thinking. Having a friend for support can help to alleviate some of their fears.

To further include all students, Mrs. Malloy utilized a variety of instructional strategies. In order to meet the diverse needs of students in her class, Mrs. Malloy thought in advance about the assignment and its prerequisite skills. A component of the differentiation process involves breaking down the concepts and skills and making them accessible to everyone in her class. For example, she anticipated students having some difficulty with how to begin the task, so she had the students model the activity; she also was available to provide support in areas such as determining proportions or lines of best fit when the students needed it. Mrs. Malloy provided the instruction that was needed to whoever needed it in an alternative way. She did not adapt the curriculum; she adapted her instruction to the curriculum. The following section describes some more possible adaptations for students.

**This type of teaching does not occur without thoughtful preplanning!**

## ADAPTATIONS AND EXTENSIONS

This activity can be adapted and modified in a variety of ways and may be used with students in sixth through ninth grades or anyone taking Algebra I. Students could have the data provided for them, or they could collect data together as a whole class (rather than in small groups), similar to the way Mrs. Malloy had her students model it at the beginning of the lesson. When the entire class uses the same data set, the answers should end up being the same rather than spread across the wide range of acceptable answers arrived at by groups who collect their own data. (Note that if groups collect their own data, it more accurately represents the real life situation in that the answer will be different each time.)

Younger students who are not yet graphing linear functions can investigate the activity using proportional reasoning only. Conversely, this may be adapted as an algebraic investigation exclusively. Students could be directed to use either a high- or low-tech method of finding the line of best fit: graph paper, graphing calculators, or graphing software.

This activity may be modified to be more challenging and to include higher-order thinking. Students could be asked to determine when the handshake would have to begin on the east coast to end on the west coast at precisely midnight of December 31. Students could also be asked to make comparisons of the two methods: proportional reasoning and algebraic reasoning.

This activity could also be turned into a cross-curricular project with connections to social studies and economics. In Chapter 6 we explore this idea in more detail.

We now move on to the next example—surface area in the seventh grade.

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### VIGNETTE 2: SEVENTH GRADE— SURFACE AREA WITH POLYDRON SHAPES

*Mr. Romo is preparing to teach surface area to his seventh grade class. This is his eighth year teaching seventh grade, and he has noticed similarities in his students' understandings from year to year. Most of his students remember the formulas for the area of basic two-dimensional shapes (rectangles and triangles). They experience difficulties with more complicated shapes, however, since they do not visualize how more complicated shapes, like trapezoids, can be created by combining basic shapes.*

*He has spent time earlier in the year using hands-on activities to give meaning to the area formulas for parallelograms and trapezoids based on rectangles and triangles. One lesson that he found especially valuable to his students involved using geoboards to create shapes. Students were able to use rubber bands to divide the*

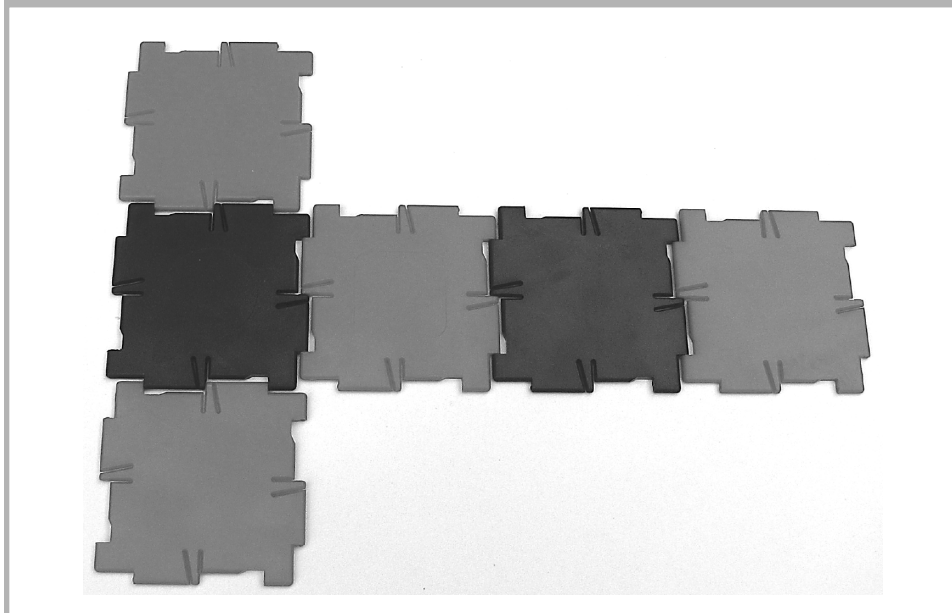


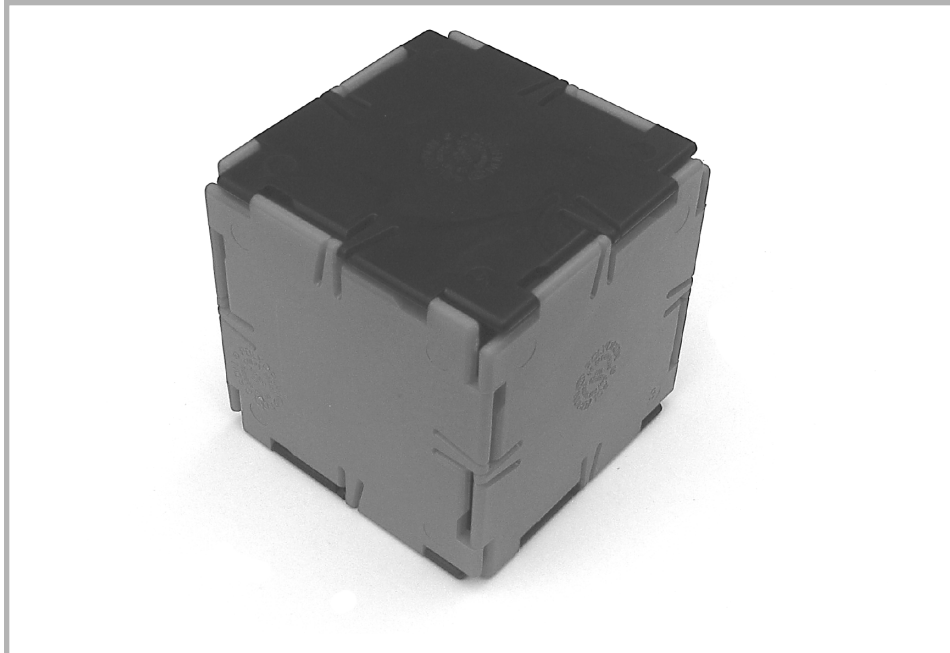
*new shapes into rectangles or triangles to find the total area. In his lessons on surface area, Mr. Romo plans to use a similar hands-on approach so that students can construct their own understanding of surface area. Students will use commercially available manipulatives called Polydron shapes to create different three-dimensional figures, and then they will take their figures apart to determine that the surface area is composed of familiar two-dimensional shapes.*

*Mr. Romo begins the activity by reviewing the area of two-dimensional figures. He asks his students to describe how to find the area of several figures that he has drawn on the board. He then begins to develop the concept of surface area of a three-dimensional object by talking about wrapping presents with wrapping paper. He introduces the Polydron shapes by showing six squares snapped together to form a flat “net” for a cube (see Figure 1.2). Students determine together that since the side length of the square is 1 unit, the surface area of the net is 6 square units. When Mr. Romo folds the sides up and snaps them together to form a  $1 \times 1 \times 1$  cube, students are satisfied that they can “see” that the surface area of the cube is still 6 square units (see Figure 1.3).*

*Once Mr. Romo has finished with his introduction to the activity, he divides students into groups and gives each group a packet of Polydron shapes (squares, rectangles, and triangles; hexagons could be used later for a challenge) and asks them to investigate the types of prisms that can be made with the two-dimensional pieces when they are snapped together. Students take about five minutes to put together various prisms and become familiar with the manipulatives. Each group of students is then directed to find the surface area of the different prisms they have created.*

**Figure 1.2** Polydron Net for a Cube



**Figure 1.3** Polydron Cube Formed From Net

Students take about 30 minutes to find the surface area of their prisms. Mr. Romo brings the class back together for discussion and asks each group to tell the class what the surface area is for one of the figures they have assembled. When students are finished with their presentations, Mr. Romo asks the class to summarize their findings by giving a general description of how to find the surface area of a prism. This part of the lesson is difficult for many students, but he wants to give them opportunities to think about the patterns found in the prisms they have investigated. He encourages his class to give a verbal description of the formula for surface area as well as an algebraic description using variables to represent the dimensions of the sides of the prisms.

At the end of the period, Mr. Romo wants to lay the foundation for the lesson the following day. He plans to continue the investigation of surface area of prisms and challenge his students to make algebraic generalizations based on observed geometric patterns. He poses the following question for students to begin thinking about for the next day:

*Given a square prism (a cube), how does the surface area of the prism change if each side dimension is doubled? Tripled? Quadrupled? Can you find any patterns in your answers?*

The next day, students begin class by discussing this problem with their group members. They have lots of ideas about how to begin. One group decides to enlist

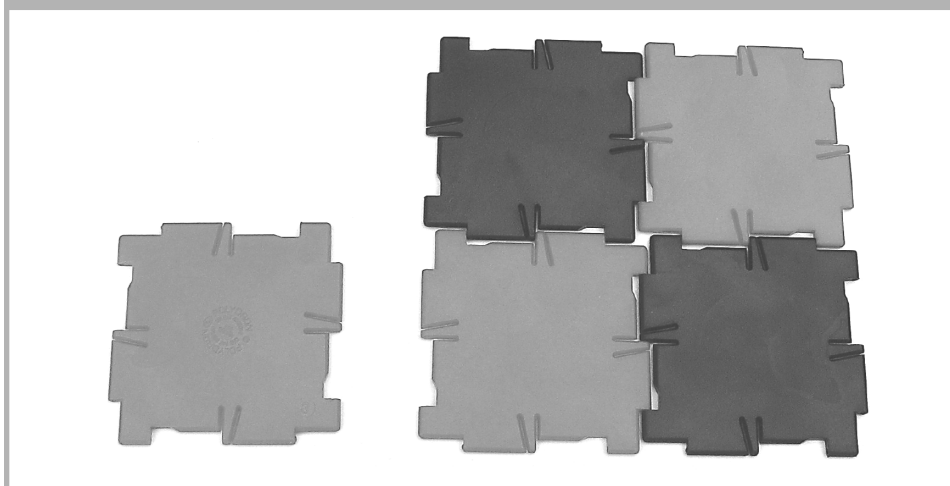
the help of another group, and they each agree to create two different figures with the Polydron shapes and then share their created figures with each other. They quickly realize that while it is fairly straightforward to create a cube with six Polydron shapes that are one square unit each, and not too much harder to double the side dimensions, it is much more difficult to create shapes that are triple and quadruple the original side lengths, because the sides become much bigger and harder to work with (see Figure 1.4).

At this point a bit of frustration begins to set in. Mr. Romo observes this minor setback and resists the urge to help these two groups out. He has learned from prior experiences that students develop stronger understandings of mathematics when they work to figure out their difficulties without his help. He realizes, however, that there is a fine line between the right amount of frustration and too much frustration, which will lead his students to give up. And although he wants his students to construct their own learning, he does realize that some need a bit of help to get them to the next step.

Just as he begins to walk closer to the two groups, he overhears one student say, "This is too hard to put together! I am going to draw it instead." The student takes out a sheet of graph paper from his binder and draws the two-dimensional sides that make up the prism that they are investigating. He uses one square unit for the original side length, and then doubles, triples, and quadruples all the sides. Other students begin to follow his lead, and several chime in at once to help him figure out the new areas of each of the sides that make up the cubes. Since they have chosen to create square prisms with dimensions  $1 \times 1 \times 1$ ,  $2 \times 2 \times 2$ ,  $3 \times 3 \times 3$ , and  $4 \times 4 \times 4$  units, their surface areas are 6, 24, 54, and 96 square units respectively.

They stare at their answers, and cannot come up with any patterns. Mr. Romo suggests to one group that they organize all of their findings in a table in order to

**Figure 1.4**  $1 \times 1$  and  $2 \times 2$  Units Created From Polydron Shapes



*look for patterns. Since this strategy is one they have used in class successfully before, they are happy with this suggestion and begin creating the table. Another group spends some time looking at the nets they have drawn on paper, and these students notice that doubling each square produces an area that is bigger than the original area. Mr. Romo is happy that they have used their nets to help them find a pattern, and he encourages them to determine how much bigger the new area is than the original area.*

*By the end of class that day, most groups have come up with some patterns that they have found. Each group is given some time to share their findings with the class. Mr. Romo makes sure to ask each group to explain how they found their patterns, since some students created tables to help the patterns become more apparent, while others used the Polydron shapes or the nets they had drawn to help them compare the surface areas. One student was actually able to demonstrate his finding that when you double the dimensions, the surface area is four times the original area, and when you triple the dimensions, the surface area is nine times the original area. (See Appendix B for the table and algebraic generalization.)*

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## DISCUSSION

In this section we will focus on the following points:

- ❖ Mathematical Connections
- ❖ Higher-Order Thinking Skills
- ❖ Facilitating Communication
- ❖ Engaging Learners in High-Quality Mathematics
- ❖ Adapting Instruction to Include All Learners

This vignette, like the first one, illustrates a lesson plan implemented by an experienced teacher, one who pays close attention to the many different factors that make up a successful learning experience. In this lesson, seventh grade students learned about surface area of prisms in a hands-on manner. Mr. Romo used his experience teaching the concepts of area and perimeter and extended it to measurement of three-dimensional figures. He was able to make mathematical connections come alive in a well-chosen activity that lent itself well to both algebraic and geometric investigation.

Notice that to set up the lesson for the second day, Mr. Romo posed a higher-order question to his class—that of determining the change in surface area when dimensions are increased. Students were asked to investigate this question and come up with patterns that they found. This question was given to the class to encourage deeper thinking about the concepts, yet it was posed generally enough to allow students to experience success at

many different levels. This critical part of the lesson ensures that students are challenged beyond understanding of procedures and rules and are thinking more deeply about the concepts involved. Yet it is a component often overlooked by teachers who are simply trying to get through the text.

Communication was also a critical component. Plenty of small-group as well as whole-class discussion took place throughout the lesson. One of the great strengths of this activity lies in the rich discussions that took place during the group interactions. Notice that Mr. Romo did not spend the whole class period at the board doing all the work. Rather, he involved the students in their learning through active investigation and class discussion. Mr. Romo stepped back from the traditional role of teacher as lecturer and took on more of a facilitator role. He skillfully used classroom discourse to promote active learning by encouraging his students to reason through the questions themselves and construct their own understanding.

Mr. Romo's goal is to engage all of his learners in high quality mathematics, and therefore, he is always thinking of how to adapt or modify lessons to make them accessible to all of his students. Note that there were a variety of instructional methods used in order to include all of his learners, even those with special needs. English language learners' and special needs students' specific needs were addressed by using strategic groupings and multiple avenues to see the mathematics in different ways. He used various teaching techniques, such as active participation in solving problems, concrete and pictorial representations of surface area, and verbal discussion of the results. The first way he often introduces a concept is with a concrete representation. This lesson was no different; he used the Polydron manipulatives as a means for students to construct their own understanding of area and surface area. He is also aware of the importance of students verbalizing and clarifying their mathematical thinking in writing. This process benefits all learners, even students who may have learning difficulties or are learning English as a second language. Finally, he hopes that by engaging his students in activities such as this one, they will be more inclined to enjoy mathematics, have a positive attitude toward learning mathematics, and have more confidence in their own mathematical ability.

## **ADAPTATIONS AND EXTENSIONS**

The use of Polydron shapes makes this activity engaging due to the hands-on nature of the manipulative. Polydron shapes are easy to snap together, so students can quickly create many three-dimensional objects for investigation. However, if Polydron shapes are not available, this activity can be adapted by using card stock or graph paper to create nets for the prisms that are being investigated. Students can fold up the nets and secure them with tape.

Students in Mr. Romo's class can continue to study surface area for several more days, investigating other shapes such as rectangular and triangular prisms, pyramids, and cylinders. Students who need a challenge could be asked to investigate more complicated shapes, such as hexagonal or octagonal prisms.

## SUCCESS IN TEACHING MATHEMATICS

In these two examples, both Mrs. Malloy and Mr. Romo demonstrate commitment to high-quality mathematics instruction for their students. Many of the strategies they implemented have been identified by experts as best practices in teaching mathematics. On the basis of current research and beliefs on the teaching and learning of mathematics (Grouws & Cebulla, 2000; National Association for Gifted Children, 2005; Tucson Unified School District, n.d.; National Mathematics Advisory Panel, 2008), there is general consensus that the following statements are in line with best practices in teaching mathematics:

- ❖ Meaningful mathematics should be taught in a problem-solving environment that balances both conceptual and procedural understanding of mathematics.
- ❖ All students should be given the same opportunities to learn high-quality mathematics.
- ❖ Communication, both verbal and written, should be a means to facilitate students' reflection and clarification of their own understanding.
- ❖ Students should be engaged in constructing their own learning.
- ❖ Mathematics should be presented in a developmentally appropriate manner, using a variety of instructional methods and suitable support, such as technology and manipulatives.
- ❖ Assessment for both instruction and evaluation should be an integral part of instruction.
- ❖ Attention to beliefs and attitudes related to learning mathematics should be addressed throughout instruction.

## SUMMARY

This chapter has provided a glimpse of what some of the many facets of successful teaching and learning of mathematics look like. The two examples describe classrooms in which teachers are flexible, listen carefully to their students, and adjust the discourse to support student progress and success. The importance of these features will be discussed in detail throughout the book so as to further support a new teacher's ability to master them in the classroom.