

Preface

S*trategies for Teaching Whole Number Computation* provides a practical intervention model with targeted hands-on materials for teachers to use when working with struggling learners. This book is a comprehensive resource—providing the “what,” the “why,” and the “how”—to guide and support classroom work with whole number computation *at whatever grades students may need intervention* for these skills and concepts.

This book addresses key concepts of the *Number and Operations* Content Standard recommended by the National Council of Teachers of Mathematics (NCTM; 2000) in their *Principles and Standards for School Mathematics*. In 2006, NCTM expanded upon those standards with its publication *Curriculum Focal Points for Prekindergarten Through Grade 8 Mathematics: A Quest for Coherence*. That document describes a new approach to curriculum development that focuses on areas of emphasis within each grade—recommending the most significant mathematical concepts and skills that should be taught at each grade level. *Strategies for Teaching Whole Number Computation* addresses the depth of coverage envisioned by the focal points with respect to number and operations for whole numbers for grades two through five.

“Historically, number has been a cornerstone of the mathematics curriculum.”

—NCTM (2000, p. 32)

NCTM Content Standard:

Number and Operations

Instructional programs from prekindergarten through grade 12 should enable all students to—

- understand numbers, way of representing numbers, relationships among numbers, and number systems;
- understand meanings of operations and how they relate to one another;
- compute fluently and make reasonable estimates.

—*Principles and Standards for School Mathematics* (NCTM, 2000)

NCTM Curriculum Focal Points:

Number and Operations—Whole Numbers

Grade 2: Develop quick recall of addition facts and related subtraction facts and fluency with multidigit addition and subtraction.

Grade 3: Develop understandings of multiplication and division and strategies for basic multiplication facts and related division facts.

Grade 4: Develop quick recall of multiplication facts and related division facts and fluency with whole-number multiplication.

Grade 5: Develop an understanding of and fluency with division of whole numbers.

—*Curriculum Focal Points for Prekindergarten Through Grade 8 Mathematics: A Quest for Coherence* (NCTM, 2006)

Communication

“Communication is an essential part of mathematics and mathematics education.”

—NCTM (2000, p. 60)

Reasoning and Proof

“Teachers can understand students’ thinking when they listen carefully to students’ explanations.”

—NCTM (2000, p. 126)

Representation

“Representing numbers with various physical materials should be a major part of mathematics instruction in the elementary grades.”

—NCTM (2000, p. 33)

Content Knowledge

Knowledge of the subject

Pedagogical Knowledge

Knowledge of instructional practice

Pedagogical Content Knowledge

Knowledge of mathematical content for its use in the classroom

—Schulman (1987)

The instructional strategies described in this book are pedagogically aligned with all five NCTM process standards: problem solving, reasoning and proof, communication, connections, and representation. The following three process standards are especially embedded in the Intervention Activities of this book:

- *Communication*: Suggested questions that may be used directly with students are provided throughout the Intervention Activities; suggestions for student writing opportunities are also provided.
- *Reasoning and Proof*: Many of the suggested questions, along with the writing opportunities, require students to *explain their reasoning and thinking*.
- *Representation*: The concept development in the Intervention Activities utilizes manipulatives, number lines, grids, and other models and diagrams.

Three types of knowledge crucial for teaching mathematics at any level are described at the left. O’Donnell (2009) citing Hill and colleagues (2004) notes that teachers illustrate *pedagogical content knowledge* by “generating representations, interpreting student work, and analyzing student mistakes.” *Strategies for Teaching Whole Number Computation* addresses all three types of knowledge in a concise, user-friendly way. With a focus on *how* to apply specific pedagogy to specific content, this resource is especially strong in the area of pedagogical content knowledge. Because this book develops *mainstream mathematics concepts in a way that is truly meaningful for students*, teachers should find this book to be an effective supplement to *any* textbook program—from those on the traditional end of the continuum to those that are based on reform.

RESEARCH BASE FOR THE BOOK

Strategies for Teaching Whole Number Computation is informed by academic research conducted and analyzed during the past 35 or more years. In developing this book, the author provides a mix of this extensive research base with his personal experience of about 40 years in mathematics education. The author’s experience includes teaching at various levels, including classroom (grades five through eight), community college (in a remedial teaching lab), and university (methods courses).

Although some errors that occur in students’ work with whole number computation are due to incorrect recall of number facts or carelessness,

many are due to misconceptions and the use of incorrect strategies. According to Pincus and colleagues (1975), “too often, when teachers find errors in a child’s work, they mark the example wrong, assume that the child did not master the basic facts, and prescribe further drill. *Careful analysis of errors through observation and interviews with individual children is essential*” (p. 581).

A key premise of this book is that if teachers (1) analyze student work for *error patterns* (revealed through diagnostic tests, practice, activities, and student discourse and oral interviews) and (2) then provide timely, targeted, and meaningful intervention, student errors will decrease in frequency—while at the same time student understanding of concepts will increase. By taking into account academic research on how students learn, the intervention strategies illustrated in this book should result in improved student performance and more positive student dispositions toward learning mathematics.

HOW THE BOOK IS ORGANIZED

The book begins with a substantial section, “A Look at the Academic Research: Intervention in the Mathematics Classroom.” This academic research supports the goals and premises of the book, the pedagogical practices utilized in the Intervention Activities, and other aspects of intervention. These practices include activating prior knowledge, scaffolding, using representations, using estimation and mental math, introducing alternative algorithms, differentiating instruction, participating in instructional games, and more.

Next is a two-part section titled “Big Ideas in Computation and Problem Solving.” The first part provides the principles on how our base-ten place-value system underpins multidigit computation. Much of this book’s intervention work is based on those principles. The second part of this section is titled “Actions and Operations: Problem Structures for Addition, Subtraction, Multiplication, and Division.” Here, illustrative models are provided for teachers to use with children to help them decide which operation to use to solve a given problem by thinking of *actions* that can be done with objects (or other representations) that relate to mathematical operations. By focusing on real-world contexts for the operations, key connections are made between *problem solving* and computation.

According to Kilpatrick, Swafford, and Bradford (2001), “Studies in almost every domain of mathematics have demonstrated that problem solving provides an important context in which students can learn about number and other mathematical topics” (p. 420). The author suggests that the problem structures described in this section be used with students as they study each operation. Students should then be asked to write their own word problems based on these structures to enhance their understandings of the operation. Because problem solving is not the major focus of this book, teachers are encouraged to integrate additional resources for problem solving when they teach computation.

“Basic skills with numbers continue to be vitally important for a variety of everyday uses. They also provide crucial foundation for the higher-level mathematics essential for success in the workplace which must now also be part of a basic education.”

—Lowenberg Ball et al. (2005, p. 1056)

“A curriculum built on focal points . . . has the potential to offer opportunities for the diagnosis of difficulties and immediate intervention, thus helping students who are struggling with important mathematics content.”

—NCTM (2006, p. 5)

“The draft *Common Core State Standards for Mathematics* endeavor to follow such a design [for coherence], not only by stressing conceptual understanding of the key ideas, but also by continually returning to organizing principles such as place value or the laws of arithmetic to structure those ideas.”

—*Common Core State Standards for Mathematics* [draft], National Governors Association and the Council of Chief State School Officers (2010, p. 3)

Four main units then follow—one each for addition, subtraction, multiplication, and division of whole numbers. The units follow a predictable format. Each begins with a diagnostic test (in multiple-choice format), followed by an Item Analysis Table that keys student incorrect test responses to specific error patterns. (Each distractor on the tests is based on a specific error pattern.) A detailed section of error patterns with *step-by-step* Intervention Activities (the heart of the book) then follows. Initially for each operation, the Intervention Activities support student success with the “traditional algorithm” for that operation. However, because historically many students struggle with some of the traditional algorithms, instruction is also provided for the use of *alternative interventions* or *alternative algorithms*. It should be noted that the Intervention Activities (both for the traditional and for the alternative algorithms) may be used with students as part of the *initial instruction*—and are not just intended for use with students *after* they may have struggled with a concept.

A hallmark of this book is its strong focus on *teaching for understanding* in developing the Intervention Activities for each operation. Guided questions (with suggested student responses) and hands-on experiences—all with an emphasis on *place value* and the properties of operations—are used to achieve student understanding. It should be noted that the draft *Common Core State Standards for Mathematics* (2010) likewise places an emphasis on place value and the laws of arithmetic. Following the Intervention Activities for each unit is a short section of practice (keyed to the Item Analysis Table). Supplemental material (blacklines) for estimation, instructional games, and follow-up activities are also included for each unit.

Each main unit, along with the sections on academic research and “Big Ideas,” concludes with a set of *Questions for Teacher Reflection*. These open-ended questions are intended to provide springboards for discussion among teachers or preservice teachers who may be using this book in a professional-development setting, workshop, or methods course.

INTENDED USES AND AUDIENCE FOR THE BOOK

Strategies for Teaching Whole Number Computation is intended to serve a wide audience of educators, and there are a variety of ways to use the book.

- **Intervention program for whole number computation in pullout or full-classroom situations:** The book may be used as a full-fledged intervention program for whole number computation. The book provides a comprehensive tool for quickly diagnosing and pinpointing trouble spots students are encountering—with specific Intervention Activities tailored to address the weaknesses identified.
- **Resource for elementary classroom teachers:** The book provides the types of errors struggling students frequently make with whole number computation. Teachers, especially those with limited experience teaching this content, should find this knowledge valuable as they teach computation concepts.
- **Resource for special education teachers:** These teachers—from the elementary level through high school—should find this material useful as they work with students individually or in small groups.
- **Resource for teachers and tutors working in developmental labs at the high school and community college levels:** The book provides fresh approaches to help students who have been struggling for years to learn how to compute.
- **Resource for instructional supervisors and curriculum coordinators:** These educators should find this book to be an important tool to use as they work with their teachers.
- **Text that may be used with teachers and preservice teachers in mathematics methods courses, workshops, and professional development programs (including online programs):** Educators at all grade levels should embrace this book as a key source of pedagogical content knowledge that they can directly use in their classrooms. The *Questions for Teacher Reflection* provide opportunities for teacher discussion and/or assignments in such settings.
- **Resource for educators interested in academic research on intervention:** This book provides a handy compilation of important research findings over the past 35-plus years related to intervention for whole number computation.

