1

Equivalent Fractions and Partitioning Sets

Keys to Success in Higher-Level Mathematics, Grades 6–7

Helen Weingart

INTRODUCTION TO THE UNIT

Proficiency with fractions is an important foundation for learning more-advanced mathematics. My own experience and conversations with teachers of mathematics, including teachers of Advanced Placement calculus and statistics, indicate that fractions are a major source of confusion, error, and misconceptions for our students. This unit uses geometry as a tool to eliminate these areas of confusion and promote an understanding of fractions. Geometry is a unique but accessible way to develop an understanding of a range of fraction concepts; also, it connects with a core component of calculus.

This unit focuses on two major concepts that span several areas in mathematics. These concepts are equivalence and rate of change.

The idea that things can be partitioned or split into parts of equal size underpins the concept of fractions. Being able to partition into any number of parts is the significant link between multiplication, division, and fractions, and fractions are reciprocal with geometric understanding. Students often lose sight of the equivalence concept when partitioning different geometric shapes. They tend to focus on the "size" rather than the "amount" of the region.

Although students often can be taught fairly quickly to produce equivalent fractions by rote, they usually have little understanding of what they are doing or why.

A process based on extensive experience with partitioning quantities by physically or mentally repartitioning materials builds the understanding of what equivalence of fractions means. The goal is to teach students to visualize fractional parts, and later teach them to generalize to a technique for producing equivalent fractions by computation when visualizing is difficult.

The second area of focus in this unit is the concept of rate of change. This is a novel way to look at fractions; it leads directly to a fundamental understanding of linear functions in algebra. Students need to extend their understanding of fractions insofar as it can represent rate of change. Students can algorithmically state that 1 of 4 cookies is equivalent to 2 of 8 cookies and 4 of 16 cookies, and so on, using a "double the numerator, double the denominator" rule. However, should you ask them to complete an equivalence using 4 of 16 with 3 of x, they are perplexed. In explaining to students that the original fraction could be represented on the coordinate plane (up 1 over 4), they find that they can generate a myriad of equivalent rates while projecting denominators if they go up $\frac{1}{2}$ then over 2. This connection to algebra opens the door to deeper understanding as students move along in their mathematical careers.

A key instructional strategy is allowing students to develop their own algorithms. By the time we see students in middle school, they have had experiences with fractions; they may have had little opportunity to develop their own personal algorithms, however, or to extend those strategies to routine fraction procedures. My experience reflects what the research tells us: "[W]hen children are allowed to create and invent, their fertile minds enable them to solve problems in a variety of original and logical ways. When their minds have not been shackled by rules and conventions, children are free to invent procedures that reflect their natural thought processes. . . . [C]hildren can develop sophisticated and meaningful procedures in computation and problem solving without explicit instruction in the use of conventional algorithms" (Kamii and Madell, as quoted in Warrington, 1997).

Finally, a significant underpinning for success with the fraction notations in higher-level mathematics is that students understand that the fraction notation is a shorthand way to show the division sign. It is not unusual for students to struggle to work out the "answer" to $3 \div 5$, such as when sharing three chocolate bars among five friends, not seeing immediately that each must get $\frac{3}{5}$ of a bar. Geometry is a unique and effective tool that assists students in developing the number sense required to use fractions flexibly.

Students are engaged in activities that provide them with extensive experience in splitting a diverse range of discrete and continuous wholes into equal-sized parts so that they are able to construct a suitable partition even if they are not given a predrawn diagram. The goal is to provide students varied experiences with reasoning about fractions via divided quantities, numerical components, reference points, numerical conversions, and geometric representations, thus enabling students to work successfully with all these interpretations.

A final note: when moving through this unit on fractions, it is extremely important to have the manipulatives for students in place. When students are working independently, they should have manipulatives on their desks. When students are working in groups, the group should have manipulatives within reach. It is common to have manipulatives centrally located in the classroom for student use. While this certainly provides for student learning, it does inhibit students. Think of how a student would feel if he had to go over to the "manipulative center" to get the tiles or counters to complete the activity. Everyone would watch that student go and get the tiles—how embarrassing. In all likelihood, he would not get them at all. By placing manipulatives within the reach of the students, they will reach for them. In reaching for them, they will use them. In using them, they will strengthen their understanding and enhance their Advanced Placement potential.

CONTENT FRAMEWORK

Discipline-Specific Concepts

C1: Equivalence

C2: Rate of change

C3: Part-part-whole relationships

C4: Partitioning

C5: Equivalent rates of change

C6: Ratio

C7: Set interpretation

C8: Region models

C9: Complex fractions

Principles and Generalizations

P1: Fractions describe parts of a whole. The whole can be an object, a collection, or a quantity. Fractions represent parts of discrete quantities (collections of objects).

P2: The size of a fractional part is based on the size of the unit. Equal parts need not look alike, but they must have the same size or amount of the relevant quantity.

P3: The fraction notation is used to represent a rate, a ratio, and a proportional relationship.

P4: A fraction notation can represent either an additive $\left(\frac{3}{4} = \frac{1}{4} + \frac{1}{4} + \frac{1}{4}\right)$ or multiplicative relationship: $\frac{3}{4} = 3\left(\frac{1}{4}\right)$.

P5: Models help students clarify ideas that are often confused when explored in a purely symbolic form.

P6: The same fractional quantity can be represented by many different fractions.

P7: Even though set size varies, when fractions are equivalent, the rate of change is constant.

P8: Equivalent fractions are two ways of describing the same amount by using different-sized fractional parts.

P9: A fraction also represents a rate of change.

P10: Rate of change describes the relationship between two variables. It tells us the slope of the line. Slope is a very important ratio in algebra.

P11: Rate of change is used to analyze real-world phenomena, make predictions, and assess possible outcomes.

P12: A variety of models can be used to represent wholes and parts of wholes.

P13: Fraction names represent the relationship between parts and wholes.

P14: Drawings (models) do not speak for themselves. The same diagram of a divided quantity can represent many different relationships, depending on how the whole is defined. (Different amounts can represent a whole. Different wholes make different fractional parts in the same model.)

P15: Making equivalent rates of change is the foundation for the algorithm for complex fractions.

P16: A ratio is a number that relates two quantities or measures in a given situation in a multiplicative relationship (in contrast to a difference or additive relationship).

P17: Ratios compare any two amounts that can be parts of the same whole, wholes, or parts of different wholes.

P18: A fraction symbol can be used to represent a division or ratio relationship between quantities.

P19: There is a link between fractions and division. A fraction is another way of expressing division: $2 \div 3$, $\frac{2}{3}$, and $\frac{1}{3}$ of 2 all mean the same thing.

Skills

S1: Develop an intuitive sense of the magnitude of fractional numbers.

S2: Choose and describe the most efficient strategy to solve problems with commonly used fractions.

S3: Use mental math to solve simple problems with commonly used fractions.

S4: Create and describe mathematical rules (algorithms) for a wide variety of patterns.

S5: Recognize that a fraction describes the pattern that is a rate of change.

S6: Interpret rate of change from graphical and numerical data.

S7: Learn how to graphically represent rate of change.

S8: Locate, identify, and order fractions on the coordinate plane.

S9: Understand the "meaning" of numerator and denominator.

S10: Use physical objects and visual models to represent a whole when given a fractional part, or to represent a fractional part when given the whole.

S11: Develop and verbalize mathematical rules that describe patterns that represent equivalent amounts in sets of different sizes.

S12: Simplify fractions to represent rates of change.

S13: Recognize that a fraction bar is a grouping symbol: $\frac{2 + 3}{3 + 4} = (2 + 3) \div (3 + 4)$.

S14: Build models and draw diagrams to describe the relationship between fractions.

S15: Describe the difference between fractions and ratios.

S16: Decompose fractions into factors.

S17: Write numerical representations for geometric representations of fraction sets.

S18: Interpret a fraction that has a fraction in the numerator or denominator or both.

Standards

The concepts, principles, and skills included with this unit reflect the national standards adopted by the National Council of Teachers of Mathematics (NCTM). These standards are listed below:

1. Compute fluently and make reasonable estimates.
2. Select appropriate methods and tools for computing with fractions and decimals from among mental computation, estimation, calculators or computers, and paper and pencil, depending on the situation; apply the elected methods.
3. Develop and analyze algorithms for computing with fractions, decimals, and integers, and develop fluency in their use.
4. Develop and use strategies to estimate the results of rational-number computations and judge the reasonableness of results.
5. Apply and adapt a variety of appropriate strategies to solve problems.
6. Communicate mathematical thinking coherently and clearly to peers, teachers, and others.
7. Analyze and evaluate the mathematical thinking and strategies of others.
8. Use the language of mathematics to express precisely mathematical ideas.
9. Analyze functions of one variable by investigating rates of change, intercepts, zeros, asymptotes, and local and global behavior.
10. Investigate how a change in one variable relates to a change in a second variable.
11. Identify and describe situations with constant or varying rates of change and compare them.
12. Create and describe mental images of objects, patterns, and paths.
13. Use geometric models to represent and explain numerical and algebraic relationships.

14. Use geometric models to gain insights into and answer questions in other areas of mathematics.
15. Use graphs to analyze the nature of changes in quantities in linear relationships. Approximate and interpret rates of change from graphical and numerical data.

This Unit and the Parallel Curriculum Model

Within the context of the Parallel Curriculum Model, these lessons on fractions focus primarily on the Core Parallel and the Curriculum of Practice. The concepts and principles about fractions presented here are basic to the discipline, thereby aligning it to the Core Curriculum. The lessons help students answer general questions such as, What does this information mean? Why does this information matter? Do these ideas make sense in my life? Furthermore, the lessons are organized in a way that facilitates students' ability to remember, make meaning, and use what they know as they journey toward expertise in mathematics.

These lessons also reflect the Curriculum of Practice because the learning activities invite students to engage in the work of practicing professionals. Mathematicians deal with problems, determine meaningful patterns of information within the problem, seek strategies for solving the problem, and communicate their findings. Students who complete these lessons will find themselves asking questions such as, What tools and skills does the mathematician use? How does a practitioner approach problems like these? On what basis does a mathematician draw conclusions? Other, more-specific questions for these lessons on fractions are listed below.

Core Curriculum

1. What are we communicating when we use a fraction? What does it mean to have equal shares of something?
2. How can we tell if we have equal shares if the sizes of the pieces are different?
3. Why do we use mathematical rules? What do those rules tell us?
4. Is there more than one rule that could be used to describe a pattern?
5. How much is "one"?
6. What does it mean to use the fraction bar as a grouping symbol?
7. How can we (or, What does it mean to) decompose numbers using either multiplication or addition?
8. How does rate of change help us understand the world around us?
9. How does a ratio compare to fractions?
10. Is decomposing fractions the same as decomposing whole numbers?
11. How do the patterns generated by complex fractions compare to the patterns generated by simple fractions?

Curriculum of Practice

1. What is a mathematical model?
2. Why are models important in mathematics?
3. How are models used in the "real world"?
4. How can we use mental math to make complex fractions easy to understand?
5. How does rate of change help us understand the world around us?
6. Why is it important to be able to partition flexibly?

ASSESSMENTS

This series of mathematics lessons contains a matched pre- and post-assessment. Furthermore, the items on the matched pre- and post-assessment have been purposefully created to assess important concepts and understandings. The first 21 items on the pre-assessment and post-assessment measure a student's flexibility to interpret fractions. Items 1, 2, and 6, for example, illustrate traditional ways to represent fractions. Items 3, 5, and 19–21, on the other hand, are nontraditional ways, and incorporate common and complex fractional notation. These latter examples will require students to "stretch" their thinking about the meaning of fractions. Items 22–30 are designed to assess a student's understanding of equivalency. Items 34–39 elicit a student's understanding and flexibility in representing fractions within the context of geometry. Finally, Items 40–42 assess a student's emerging sense of factors and multiples.

The data collected from the pre- and post-assessment will provide teachers with evidence of overall student growth, as well as students' deepening understanding related to specific concepts. By measuring student pre- and post-data on a cluster of items such as equivalency, for example (Items 22–30), a teacher might decide that he will reteach that part of the unit or change the instructional approach next time he works through this set of lessons. Disaggregating the data by cluster provides teachers with powerful data to either maintain instructional practices or differentiate their instructional repertoire based on students' learning needs.

Besides the pre- and post-assessment, there are countless other opportunities embedded within these lessons for teachers to "check the weather" in their classrooms. Class worksheets, student discussions, and homework assignments are all critical sources of information about students' understandings and misconceptions. As such, each is an important source of formative assessment data that practitioners can use to customize instruction to the whole class, small groups of students, or individuals.

It is important for readers to note that many suggestions for differentiation are incorporated into these lessons. These suggestions are based on many years of experience with middle school mathematics students. These "general" reflections on above-grade-level learners, on-grade-level learners, and below-grade-level learners can be coupled with the reader's knowledge of her students as they progress through these lessons, as well as the specific information that each student has gleaned from the formative assessments contained herein.

UNIT SEQUENCE, DESCRIPTION, AND TEACHER REFLECTIONS

The idea that things can be partitioned or split into parts of equal size underpins the fraction concept. The ability to partition any number is the significant link among multiplication, division, and fractions, and is reciprocal with geometric understanding. Students need extensive experience in splitting a diverse range of discrete and continuous wholes into equal-sized parts. They need experiences that help them understand that the whole they are describing a part of can be an object, a collection, or a quantity.

Another key understanding students need to develop is that the same fractional quantity can be represented with many different fractions. Although students can often be taught fairly quickly to produce equivalent fractions by rote, they usually have little understanding of what they are doing or why. A process based on extensive experience with partitioning quantities by physically or mentally repartitioning materials builds the understanding of what equivalence of fractions means. The goal is to teach students to visualize fractional parts, and later teach them to generalize to a technique for producing equivalent fractions by computation when visualizing is difficult.

Finally, a significant underpinning for success with the fraction notations in higher-level mathematics is student understanding that the fraction notation is a shorthand way to show the division sign. Research tells us that this important relationship between fractions and division is often overlooked by both students and adults. Many will struggle to work out the "answer" to $3 \div 5$, such as when sharing three chocolate bars among five friends, not seeing immediately that each must get $\frac{3}{5}$ of a bar. The approach in this volume is designed to assist students to develop the number sense required to use fractions flexibly.

LESSON 1.1: EXPLORING EQUIVALENT FRACTIONS

Length: Two days

Unit Sequence	Teacher Reflections
Concepts C1, C2	There are two areas of focus to consider in this unit: equivalence and rate of change. Students are able to partition different geometric shapes but lose sight of the equivalence concept. For example, they partition a rectangle into fourths, and then similarly partition a circle into fourths. There is confusion about $\frac{1}{4}$ of a rectangle and $\frac{1}{4}$ of a circle. Students are focused on only the "size" of the region rather than the partition that $\frac{1}{4}$ of that region represents. Although it is true that $\frac{1}{4}$ of a pound is a different quantity from $\frac{1}{4}$ of a ton, the understanding of $\frac{1}{4}$ is what is germane to student conceptual development. The implications of equivalence extend beyond common-sized partitions. Middle school students often struggle when they are presented with a shape that has eight equal-sized partitions and a shape with four equal-sized partitions. Students are often perplexed if the first region has three of the eight partitions shaded and the student is asked to shade in an equivalent amount in the region with 4 partitions. Students do not "partition" the first region as $\frac{2}{8} + \frac{1}{8}$, extending their understanding to $\frac{1}{4} + \frac{1}{2}$ of $\frac{1}{4}$. Truly, this understanding is the linch pin for advanced mathematical understanding. The second area of focus in this unit is a fraction as a representation of a rate of change. This is a novel way to look at fractions, one that leads directly to a fundamental understanding of linear functions in algebra. Students need to extend their understanding of fractions in that they represent numerous interpretations (constructs), representations (models), and coding conventions $\left(\frac{5}{4},\ 1\frac{1}{4},\ 1.25,\ 125\%\right)$ and can represent a rate of change. Students can algorithmically state that 1 of 4 cookies is equivalent to 2 of 8 cookies and 4 of 16 cookies, and so on, using a "double the numerator, double the denominator" rule. However, if you ask them to complete an equivalence using a relationship such as 4 of 16 as it relates to the relationship 3 to x, they are perplexed.

Unit Sequence	Teacher Reflections
	In explaining to students that the original fraction could be represented on the coordinate plane as a variety of rates—"up 4 over 16" or perhaps "up 2 over 8," and so on—they find that they can generate a myriad of equivalent rates while projecting denominators. The accelerated learner can be challenged with the complex relationship of "up $\frac{1}{2}$ over 2," and can compare that rate of change to the original 4 of 16. These concepts, equivalence, and rate of change provide the connection to algebra and will open the door to deeper understanding as students move along in their mathematical careers.
Principles P1, P2, P6, P7, P8, P9, P10, P11	As we develop the concepts of equivalence and rate of change in students, it is important to realize that students are not necessarily "organized" in the traditional ways that teachers have come to expect. This is developed extensively in the narrative section, Day 1 and Day 2. This organizational issue is an important component in developing a conceptual understanding of fractions that goes beyond an algorithmic understanding; this understanding can best be mastered with manipulatives, working on graph paper, or both. When working with graph paper, you can give the dimensions for the region and ensure that students have the "exact"-sized region. While we can draw a square on the board that may look, in fact, somewhat rectangular, as adults, we can flexibly recharacterize that shape to be "square." Take the time to think of a student who is tentative and is not able to flexibly recharacterize that shape as a square. In fact, that student may be so distracted by the lack of precision that the lesson is lost to him. The manipulatives or graph paper, or both, can circumvent that distraction. Choose appropriate region dimensions for beginning activities so that the geometric partitions can be easily shaded and reconfigured. Activities such as these provide an opportunity for the teacher to engage in quality formative assessment of students. The ease or difficulty with which students adapt to the visual reconfiguration may redirect your focus. If students are struggling, additional time may be warranted, or a more formal process of partitioning included. Make sure that students can master the visual. Deep understanding of the reconfigurations extends to concepts involving geometric probability in the "fair, not fair" games.
Skills S1, S2, S3, S4	
Standards SD3, SD4, SD5, SD6, SD7, SD8, SD9, SD10, SD11, SD12, SD13, SD14	

<table>
<tr><th>Unit Sequence</th><th>Teacher Reflections</th></tr>
<tr><td>Guiding Questions
1. What does it mean to have equal shares of something?
2. How can we tell if we have equal shares if the sizes of the pieces are different?
3. How does rate of change help us understand the world around us?
4. Why do we use mathematical rules? What do they tell us?
5. Is there more than one rule that could be used to describe a pattern?
6. What are we communicating when we use a fraction?</td><td>It is extremely important to have the manipulatives for students in place. When students are working independently, they should have manipulatives on their desks. When they are working in groups, the group should have manipulatives within reach. While having manipulatives centrally located in the classroom for student use may provide for convenience of classroom management, it also may inhibit students. Self-conscious middle school students will not go over to the “manipulative center” to get the tiles or counters they need to complete activities. If teachers place manipulatives close to the students, they reach for them. In reaching for them, they use them, and they strengthen and enhance their understanding of the underlying concepts.</td></tr>
<tr><td>Introduction
Use Resource 1.1: Pre-Assessment: What Did You Learn About Fractions?

These lessons develop core knowledge and awareness of fraction patterns necessary for application in an algebraic context. This understanding is the precursor to “partitioning,” which is the significant link between multiplication, division, and fractions, and is reciprocal with geometric understanding. These two lessons are designed to explore equivalent fractions in sets of different sizes and to introduce students to the idea that a fraction represents both quantity and rate of change.
Students will identify fractional parts of sets and plot the unreduced fractions in given sets to form lines.</td><td>Two to three weeks ahead of time, ask students to complete the pre-assessment that is included at the end of this lesson. Remind students that they are not expected to know the answers to these questions. Instead, you will use the data from their work to tailor your instruction to their learning needs. The pre-assessment should take approximately 30 minutes. Refer to the Introduction section to this unit for additional information about the pre-assessment and post-assessment.
It is key for students to have opportunities to explore partitioning of sets into whole sets and fractional sets; this partitioning is the concept behind writing an improper fraction as a mixed number. There are two issues that come up continually in algebra and advanced mathematics: the use of the fraction bar and reducing fractions to their lowest terms. When presented with the problem of simplifying the expression $\frac{6x+2}{2}$, students incorrectly give the answer $3x + 2$ or $3x$, even going so far as to say $\frac{6+2}{2} = 5 \text{ or } 7\cdot$
Students have difficulty with the notion that the fraction bar is a grouping symbol that means the same as the parentheses in the distributive property, and with the idea that dividing by a number or an expression is equivalent to multiplying by its reciprocal. For example, $\frac{8}{4} = \frac{1}{4} \times 8$, or, in algebra, $\frac{2x-6}{2} = \frac{1}{2}(2x-6)$ are examples of using the fraction bar as a grouping symbol. The activities throughout the unit support the development of this understanding.
Another significant misconception that we often see in algebra is the expression $\frac{x+3}{x+4}$ incorrectly simplified to $\frac{3}{4}$.</td></tr>
</table>

<table>
<tr><th>Unit Sequence</th><th>Teacher Reflections</th></tr>
<tr><td></td><td>This misconception relates directly to how fractions are reduced to the simplest terms by factoring out the greatest common factor. Because students do not really understand the algorithm, they are unable to apply it to algebraic expressions.</td></tr>
<tr><td>Teaching Strategies and Learning Experiences

Day 1

Use Resource 1.2: Recognizing Equivalent Fractions: What Fraction of Each Set Is Shaded? as a warm-up for the group activities. Plot each set on a single grid, then plot A and B on a second grid using a different color for Set A and Set B. (Use Resource 1.3: Overhead Template.) Students should realize that these are all the same fraction in different forms.
Teacher Note. When the fractions are plotted in unreduced form, they will form a line with a rate of change of $\frac{1}{4}$. Use this concept when facilitating the discussion that follows.

Day 2

The second part of this lesson is a similar exploration of different equivalent fractions. It is designed for use as a small-group activity. Use Resource 1.2: Recognizing Equivalent Fractions: What Fraction of Each Set Is Shaded? If students plot the graphs on overhead transparencies (see Resource 1.3: Overhead Template), comparisons can easily be made among the different fractions.
Depending on the levels within groups, students can continue exploring the rate of change notion with a variety of options; for example, (1) fractions can be selected that are common fractions; (2) complex fractions can be selected and introduced; and (3) fractions can be selected that explore the use of negative values in the numerator or denominator, as appropriate.</td><td>On the surface, this may seem to be a very simple activity. However, it is often the simple concepts that we take for granted. The algorithm is so easy to present. The first concept is simply representing equivalent fraction pieces in each box. If you look at the worksheet, you will see that the circles are not shaded using a traditional shading pattern. This is intentional. The shading method of the circles does not seem as neat and organized as one would like.
In doing this activity, use overhead chips in two colors and ask students if they like the way the dark chips are organized. You can discuss the pros and cons of the arrangement of the dark chips. Now is a good time to ask if there are other ways to arrange the dark and light chips to make the task easier. Is it important for every child to model $\frac{1}{4}$ similarly? No—but it is important to realize that there is no correct way, as long as the students understand that $\frac{1}{4}$ is dark in each case. Spending time discussing this visual equivalency model will develop refined visual skills in students (see Item 2 under in the Content Framework that is in the Introduction to this unit). Spending time on this will ensure success for the right-hand side of the page, because some students may use a nontraditional coloring pattern.
In addressing the issue of fractional representations in the lesson, there will be more freedom for the learner to create his representation for the fraction. This flexibility allows the teacher to enhance student understanding while authenticating a child's way of knowing and learning. The algorithm is far less flexible than modeling.
As you continue with Resource 1.2: Recognizing Equivalent Fractions: What Fraction of Each Set Is Shaded?, the graphing component will come into play. You will now be graphing the fractions. You can see that only the first quadrant is provided. At this point, we are talking about fractions and their relationship to rates of change. (Depending on the level of your students, you can have a discussion about moving into Quadrant III, including the concept of domain; this is a perfect opportunity to differentiate for the accelerated learner.)
In workshops with teachers, the graphing of a fraction appears to be counterintuitive to their understanding.</td></tr>
</table>

Unit Sequence	Teacher Reflections
A separate student worksheet is included to use for individual work. See **Resource 1.4: What Fraction Is Shaded?**	The focus of the plotting of points is designed to develop slope. The *y*-axis is deliberately set as the numerator value and the *x*-axis is set as the denominator. The grids are labeled accordingly, so as to avoid any errors in plotting the fraction. After students create their graphs, use the following questions to guide the students to the understanding that a fraction is also a rate of change. 1. Looking at the points in Set A, how can we get from one point to the next? Is there a pattern? 2. Is that pattern the same for Set B? How are they the same? How are they different? Indicate to the students that change is moving up or down, and that the change is constant. Change represents what is shaded. A fraction is *also* a rate of change. This is different from representing an equal share of something. Understanding that a fraction is also a rate of change will help students be successful in higher-level mathematics. There is also the opportunity to discuss "up and over to the right" relating that to "down then left." This can be a rich discussion for the class: integers and fractions as division. **Resource 1.4: What Fraction Is Shaded?** can be group work or individual work as is appropriate for the class or individual students. The first two pages are simply a review. The third page is the crucial piece, and time should be spent going over those answers. There may be some difficulty in developing an algebraic rule. Use an intuitive approach; that approach is the emphasis for the entire unit. Depending on the group, you can begin discussions such as $\frac{1}{3} \cdot x$, where x represents one $\left(\frac{1}{1}, \frac{2}{2}, \frac{3}{3}\right)$. Since this is no longer the algorithm for multiplication, it now represents what is happening on the grid, so students can internalize the notion of equivalency, and thus develop understanding. This is also a connection for the multiplication algorithm. It is also appropriate to incorporate vocabulary such as the multiplicative identity element. Students can relate fractional equivalency to the fact that they started with one fraction, then generated a second fraction by using a rate of change, then a third, and so on. Students now have an opportunity to discover fractional equivalency using a nonlinguistic representation that uses a rate of change model (i.e., slope). Depending on the group, you may wish to talk about representing one as $\frac{1}{2}$ over $\frac{1}{2}$ to see what they will propose as a solution. Their answers can be "checked" using the grid. This is a very concrete way to develop the

Unit Sequence	Teacher Reflections
	notion of complex fractions that routinely occur in upper-level mathematics. **Resource 1.5: Fractions and Rates of Change** is designed to be a homework sheet, reinforcing the skills, concepts, and discussions that are developed during the lessons.
Closure Students work in small groups to summarize their responses to questions on **Resource 1.4: What Fraction Is Shaded?** Groups will be assigned to report their results for fraction sets A, B, and C. Review with students the connection between the rate of change and a fraction: • What fraction of each set is shaded? • What fraction of the set is not shaded? • What is the rule you used for determining the shaded part? • What is the rule you used for determining the unshaded part? • If you combine the shaded and unshaded parts, what is the result? • What is the rate of change?	

LESSON 1.2: EXPLORING FRACTIONAL PARTITIONS

Length: Two days

Unit Sequence	Teacher Reflections
Concepts C3, C4, C5	
Principles P1, P3, P4, P5, P6, P12, P13, P14, P15	As fraction understanding emerges, we move to a real-world understanding of fractions and partitioning. Repeated opportunities for modeling the partitioning will provide the scaffold for learning that heretofore has been overlooked in middle school. It is very easy for the teacher to move to the algorithm, and arguably even easier to just show it. I have been surprised to see that students who know the algorithm can still struggle to model the mathematics of that algorithm. In all cases, be sure that the students model all the situations. The second part of this lesson requires a deep understanding of fractions; modeling in this second part is key.
Skills S5, S6, S7, S8	
Standards SD1, SD3, SD4, SD5, SD8, SD10, SD11, SD15	
Guiding Questions 1. How does what we know about basic number relationships (e.g., the same number can be represented in many different ways: 6 + 3 = 9, 5 + 4 = 9, and so on) help us understand fractions? 2. Why do we need to be able to break a group into different-sized pieces? 3. How much is “one”? 4. What does it mean to use the fraction bar as a grouping symbol?	

Unit Sequence	Teacher Reflections
Background Overview *Exploring Fractional Partitions Using Lemon Cookies* Students are introduced to partitioning using fractions (sets) that are familiar to them. They translate the visual representation into complex fractions and move on to investigate the rate of change for the complex fractions. *Teacher Note*. Making equivalent rates of change is the foundation for the algorithm for complex fractions.	Surprisingly, teachers I have worked with find this to be a challenging activity. Their tendency is to immediately use the algorithm without modeling. When they are directed to use modeling, they are tentative when they get to Question 3 of **Resource 1.6: Lemon Cookies**. Teachers need time to discover that they can "cut the cookies in half." Once this is discovered, the discussion needs to focus on the part-whole component essential to fraction understanding. Additionally, the teachers are further consternated with Example 4. Again, further development of the part-whole relationship is the emphasis for the lesson.
Introduction Students explore partitioning sets by starting with a hands-on exploration in a familiar context: eating cookies. The focus of this activity is on the problem-solving process, not on the algorithm. **Resource 1.6: Lemon Cookies** should be used as a whole-class exploration. **Resource 1.7: Our Favorites: Chocolate and Vanilla Cookies** can be used as a group assignment (for all students) or as an individual assignment.	Refrain from using this activity as a worksheet for multiplication. Using the worksheet in that fashion results in a 10-minute task for students that simply requires the application of the algorithm for multiplication. Modeling is the key here. Students at both ends of the spectrum need a modeling experience. Students at the lower level comfortably complete these problems, with some students at the upper end struggling to create a model. My experience is that some students who know the algorithm cannot model the situation—after all, these are the very same problems that had teachers stumped. This is an indicator that those students have not internalized a true understanding of fractions.
Teaching Strategies and Learning Experiences *Day 1* It is important that each student have 18 paper circles and scissors **(Resource 1.8: Cookies Template)**. To guide students through the activity, start with the thirds.	This activity, **Resource 1.6: Lemon Cookies**, is designed to provide an option for novel problem solving with fractions. For this activity, the students have a single group—lemon cookies—with which to work. It is crucial that students have disks or use the cookie cutouts for this activity **(Resource 1.8: Cookies Template)**. It is the manipulation of the "cookies" that will develop the deep understanding that students need to progress in mathematics. The teacher needs to have "overhead" disks to model answers and solving strategies for the students. I would suggest that you bring in envelopes so that the "cookies" can go into the envelopes for later use.

Unit Sequence	Teacher Reflections
Day 2 **Resource 1.7: Our Favorites: Chocolate and Vanilla Cookies** can be done as a class. It is important that students see that each box represents one-third of a whole even though the picture may lead students to think they are dividing the bars into eighths, fifths, and thirds, respectively. Make sure the students have 12 disks representing vanilla and 24 disks representing chocolate **(Resource 1.8: Cookies Template)**. Be sure *all* students have the circle models and scissors to aid in their understanding.	During instruction, use the terminology "groups of" rather than "packs," because the idea of packs distracts from building the understanding that we would like the students to acquire. As you move through the unit, you can develop an "algorithm" so that you can express the two-thirds as $\frac{2 \textit{ groups of } 6}{3 \textit{ groups of } 6}$ so that students can see $\frac{2 \cdot 6}{3 \cdot 6}$ as $\frac{2}{3}$. You can continue this modeling through the other examples. As indicated earlier, it is crucial for advanced mathematical thinking that students understand the difference between $\frac{6+6}{6+6+6} \neq \frac{0}{6}$, which is a typical error when students see the rational expression versions of this fraction concept. This development can occur during the Lemon Cookie activity. As you continue with the lesson, students may view **Resource 1.7: Our Favorites: Chocolate and Vanilla Cookies** as being more challenging. The students must use the cookie cutouts **(Resource 1.8: Cookies Template)** to model the situations to arrive at the answers. Students can be quite creative in their representations. Allow students to model their solutions on the overhead using overhead disks in two colors. There are times when this "sharing" of solutions is short-circuited. It is important to take the time so that all the solutions are modeled, particularly if students have represented the solutions differently. This provides the opportunities for the visual and kinesthetic learner to achieve understanding.
Closure Students complete **Resource 1.9: Tiling the Family Room** for homework. Again, have students summarize for the class. The student-created problems will synthesize their understanding. Ask the students to create examples that are different from the cookie and tiles examples.	**Resource 1.9: Tiling the Family Room** is designed as a homework sheet for the students. The challenge question is to have the students design their own examples. This is a time to talk about factors, multiples, prime numbers, abundant numbers (factors of the number add to a value greater than the number), and deficient numbers (factors of the number add to a value less than the number) as appropriate with your students. This vocabulary should be integrated into the explanations the next day. Students will be developing algorithms without any "real" lesson for algorithms. They just will apply logical reasoning, and will then own the algorithm.

LESSON 1.3: FRACTURED FRACTIONS

Length: Two days

Unit Sequence	Teacher Reflections
Day 1: **Resource 1.10: Modeling Fractions**, and **Resource 1.11: Working With Fractions** *Day 2:* **Resource 1.12: Fractured Fractions and Beyond**	Teachers elicited surprise at the focus on complex fractions. I have been asked, "When will students use this?" I found this to be a very interesting comment. Complex fractions occur every day—"I traveled for half an hour at 60 miles per hour" is the easiest example to use. There is also "I lost $\frac{1}{2}$ a pound." Teachers routinely implement dimensional analysis without realizing that they have just converted their "complex" fraction into a proper fraction. In upper-level mathematics, complex fractions are the norm rather than the exception. Those fractions take on the look of one algebraic expression over another, routinely used in developing the derivative and the use of partial fractions during the early units in calculus.
Concepts C6, C7, C8	
Principles P1, P2, P11, P13, P14, P16, P17	
Skills S9, S10, S11, S12, S13	
Standards SD1, SD2, SD3, SD4, SD5, SD14	
Guiding Questions 1. How does a ratio compare to the fractions we studied in our last lesson? 2. How can we decompose numbers using either multiplication or addition? 3. What is a mathematical model? 4. Why are models important in mathematics? 5. How do models help us understand fractional relationships? 6. Is decomposing fractions the same as decomposing whole numbers?	

Unit Sequence	Teacher Reflections
Background Information	The concepts presented in this lesson are significant in the development of algebraic skills. The misconceptions regarding fractions are particularly evident. Students will routinely "cancel" numbers without regard to the fact that they can "cancel" terms only when those terms are factors. A fraction cannot be reduced by an addend in the numerator and denominator. It is not typical for students to have experiences with fractions that seem similar such as $\frac{2 \times 6}{2 \times 5}$ versus $\frac{2 + (3 + 3)}{2 + (3 + 2)}$. In an algebra class, students will routinely cancel the addend 2, due in part to a lack of understanding for the reducing algorithm. Here is an opportunity to develop understanding. Students who have worked through these problems enjoy providing lots of alternatives that would model the same "answer." A solid understanding of this concept is crucial for students.
Introduction Students have numerous misconceptions about how fractions can be combined or separated. Often in algebra and calculus, students are required to **decompose** fractions into factors in ways with which they are unfamiliar. For example, they understand that $\frac{6}{x} = \frac{2 \times 3}{x}$, but they have more trouble recognizing that $\frac{6}{x} = 6\left(\frac{1}{x}\right)$. Make sure students model the steps from the geometric to the numerical representations to eliminate fraction misconceptions. A nice extension activity would be to have students categorize the various fraction forms from **Resource 1.10: Modeling Fractions,** Question 2 (Items A–L). This activity can be diagnostic for the teacher if students simply group by operations versus value. Students should be able to create four groups: $\left(\frac{5}{3}, \frac{8}{5}, \frac{8}{7}, \frac{6}{5}\right)$. On Day 2, **Resource 1.12: Fractured Fractions and Beyond** moves from the concrete or visual understanding to the algebraic or algorithmic processes needed for Advanced Placement thinking. **Resource 1.13: Decomposing Fractions** is provided as a homework sheet.	**Resource 1.10: Modeling Fractions** seems to be quite obvious to the teacher—so obvious, in fact, that little or no time is devoted to this visual understanding of improper fractions. Students consistently make errors in this simplification process beginning in Algebra 1 continuing through to Calculus. In upper-level math, students look at expressions such as $\frac{x^2}{x^2 - 4}$ and "simplify" that expression to either $\frac{1}{4}$ or -4. I have even seen this error in calculus students: when they are working with a problem such as $\lim_{x \to 2} \frac{x^2}{x^2 - 4}$ they give similar answers: $\frac{-1}{4}$ or -4. Hence the inclusion of these worksheets. In all cases, students should model these situations. You can use the cookie cutouts from the previous lessons and model the situations with overhead counters. **Resource 1.11: Working With Fractions** moves to the conventional fraction form. It is important that students develop the algorithm: *do not* provide the algorithm for them. The worksheet, **Resource 1.12: Fractured Fractions and Beyond**, extends student understanding toward the development of algebraic representation. I would not overemphasize the variables. You will be surprised that the students will just use the variables, perhaps because there are choices available. The triangle area problem is a perfect example to use at this point. When fraction understanding is only algorithmic, students cannot "reconfigure" the algorithm to a nonconforming application. How many times have students looked at $\frac{1}{2} \cdot 11 \cdot 10$ going through in order, rather than dividing the 10 by 2 then multiplying by the 11? One could argue that students are troubled with this example, as well—$4 \times 15 \times 25$—and do not notice the 4×25 as the first choice for computing. If, in fact, your students miss this association component, take the time to develop this type of number sense with them.

Unit Sequence	Teacher Reflections
Teaching Strategies and Learning Experiences *Day 1* Using **Resource 1.10: Modeling Fractions,** make sure students model the steps from the geometric to the numerical representations to eliminate fraction misconceptions regarding simplification of fractions. **Resource 1.11: Working With Fractions** is an appropriate homework assignment. It will reinforce the concepts developed in class. *Day 2* **Resource 1.12: Fractured Fractions and Beyond** moves from the concrete or visual understanding to the algebraic or algorithmic processes needed for Advanced Placement thinking. Students will now look at fraction forms that seem quite similar: $\left(\frac{2+(3\times 2)}{2+(3\times 1)}\right)$ versus $\frac{2+(3+3)}{2+(3+2)}$. It is important to develop concrete understanding of these two concepts. Other skills that are incorporated include an extension to an algebraic understanding in differentiating: $\frac{6}{x}=6\left(\frac{1}{x}\right)$ versus $\frac{x}{6}=\left(\frac{1}{6}\right)x$. A nice extension activity would be to have students categorize the various fraction forms from **Resource 1.11: Working With Fractions,** Question 2 (Items A–L). You can use this activity as a diagnostic if your students simply group by operations versus value.	Modeling fractions as a sum in the numerator or denominator (or both) is one of those activities that can seem so basic that we tend to skim right over it. It cannot be overstated, though, that you do not have to go far to find an algebra teacher who will lament student misunderstandings. Not a day goes by without the following error: $\frac{x+2}{2+1}=\frac{x}{1}=x$. Yet every time this error occurs, the algebra teacher will ask if this is true: $\frac{7}{3}=\frac{5+2}{2+1}$? The students all agree, with an exuberant "Yes!" Then the teacher continues, and asks if this is true: $\frac{5+2}{2+1}$? = ? $\frac{5}{1}$, ? = ? 5. The students all reply, "No!" Day in and day out, teachers need to repeat this type of example. The need to own this concept cannot be underestimated. **Resource 1.11: Working With Fractions** moves to the conventional fraction form. It is important that students develop the algorithm: *do not* provide the algorithm for them. The worksheet **Resource 1.12: Fractured Fractions and Beyond** is designed to extend student understanding toward the development of algebraic representation. I would not overemphasize the variables. You will be surprised how the students will just use the variables, perhaps because there are choices available. The triangle area problem is a perfect example to use at this point. When fraction understanding is only algorithmic, students cannot "reconfigure" the algorithm to a nonconforming application. How many times have students looked at $\frac{1}{2}\cdot 11\cdot 10$ and gone through in order, rather than dividing the 10 by 2 then multiplying by the 11. One could argue that students also are troubled with this example: 4 × 15 × 25, not noticing the 4 × 25 as the first choice for computing. If, in fact, your students miss this association component, take the time to develop this type of number sense with them.
Closure	The worksheet provided moves to an algorithmic platform. It would be appropriate to ask students where mistakes could be made. Error analysis is a teaching strategy that is often overlooked. Have students correct the homework in groups. Have each group submit a problem that students consider to be a novel way to answer the questions.

LESSON 1.4: PROBING COMPLEX FRACTIONS

Length: Three days

Unit Sequence	Teacher Reflections
Concepts C2, C9	
Principles P14, P15, P18, P19	
Skills S14	
Standards SD1, SD2, SD3, SD5, SD6, SD10, SD11, SD13	
Guiding Questions 1. How do the patterns generated by complex fractions compare to the patterns generated by simple fractions? 2. How can we use mental math to make complex fractions easy to understand?	
Background Information	The unit now combines equivalence and the rate concept through the use of a geometric model. This lesson also will develop relationships between the “improper” shaded fraction, the unshaded portion of that fraction, and the resulting values.
Introduction Pass out an index card to each student and invite them to find a partner. Ask each pair to work separately at first to write down what they think a complex fraction is. Once each student has completed the task, ask them to share with each other and compare definitions. Debrief the class to find out what students’ perceptions and misperceptions about this term are.	This is an important task because it will reveal what students know and do not know about **complex fractions**, the topic for these lessons. Complex fractions are fractions that have a fraction in the numerator, denominator, or both. Many will not be familiar with the term. Others will confuse mixed numbers with complex fractions. Listen carefully for students’ misconceptions when you debrief the class. If there are a number of misconceptions, make sure to write them down so that you can correct them as this lesson progresses.

Unit Sequence	Teacher Reflections
Teaching Strategies and Learning Experiences *Days 1–3* Move on to connecting complex fractions with rate of change. Use **Resource 1.14: Rectangle Pieces.** *Teacher Note.* A grid has not been included. Use graph paper. Each problem in Set A requires a separate graph. Students may want to assign different intervals for the independent axis and the dependent axis. (Use this vocabulary to promote advanced math thinking.) Start the activity with Set A, modeling how to write complex fractions and graphing the rate of change for Sets 1–3. Make an overhead transparency to graph. Use one-inch blocks so that you can be accurate when graphing the fractional numerators. Use **Resource 1.14** with four sheets of graph paper. When graphing these, be quite demonstrative in your graphing. In Set A, Question 1, have the students go up $\frac{1}{2}$ a unit and over 1, then again up $\frac{1}{2}$ and over 1. Ask the students if there is an easier way to get to the second point. Be sure to write that if we do go up $\frac{1}{2}$ and over 1 twice, it is equivalent to going up 1 and over 2. Facilitate this discussion with the other sets: *do not* provide an algorithm (e.g., make equivalent fractions by multiplying numerator and denominator or complex fraction division). Students should do Question 4 independently. *Guiding Questions.* What fraction of each set is shaded? What fraction of the set is unshaded? Repeat the process for Set B. Add this question: Do I always have to graph this out to determine a rate of change with integers in the numerator and denominator? Have the students graph $\frac{\frac{1}{2}}{1}$, repeating that "rise" and "run" three more times. Then make a connection between rise of $\frac{1}{2}$ and run of 1, as it compares to rise of 1 and run of 2. Repeat this throughout the activity: *do not* teach the complex fraction algorithm. This concept is crucial to understanding partial fractions in calculus.	We are now connecting all the pieces for the students. It is important that the students go through the activities using the rate of change model that was used earlier in the unit. Students will start to make intuitive conversions, for example, $\frac{\frac{1}{2}}{1}$ will quickly turn into $\frac{1}{2}$. I have heard students say, "Well, if I move up $\frac{1}{2}$ a block and over 1 block you get to the point (1,2), which is $\frac{1}{2}$." They have similar connections to $\frac{\frac{1}{2}}{2}$: "This is easy, you just go up $\frac{1}{2}$ a block then over 2, which is really just $\frac{1}{2}$, and that is easy to see, because you can break up the rectangles into halves." When students have made this connection, the algorithm, while quite procedural, actually provides an answer that "makes sense" to the students.

<table>
<tr><th>Unit Sequence</th><th>Teacher Reflections</th></tr>
<tr><td>
Have students complete Set C on their own, answering the following questions in writing:

What fraction of each set is shaded?
What fraction of each set is unshaded?
What is the rule you used for determining the shaded part?
What is the rule you used for determining the unshaded part?
If you combine the shaded and unshaded parts, what is the result?
What is the rate of change?

Resource 1.15: Rectangles Break Apart can be additional independent or group work. It includes mental computation. By now, students should have developed an intuitive number sense of fractions through the use of partitioning and should be able to easily compute these problems mentally (without the need of an algorithm).
Resource 1.16: Rectangles on the Move can be used as homework or as an assessment.
</td><td></td></tr>
<tr><td>
Closure
Administer Resource 1.17: Post-Assessment: What Did You Learn About Fractions? Compare pre-assessment and post-assessment results.
</td><td>
Now is a good time to talk about the issues that arise with fractions. A fun activity is to create a fractions handbook. Have the students generate a list of common misconceptions, providing solutions that are modeled geometrically.
If you are in a school with younger students, have your class create a fractions packet for them. They could have cutout manipulatives for common fractions and word problems that can be modeled with the pieces.
Remember, you now have a group of students who have developed skill in modeling mathematical situations: do not let it end. Modeling is key to deep understanding.
</td></tr>
</table>

SUGGESTED READINGS

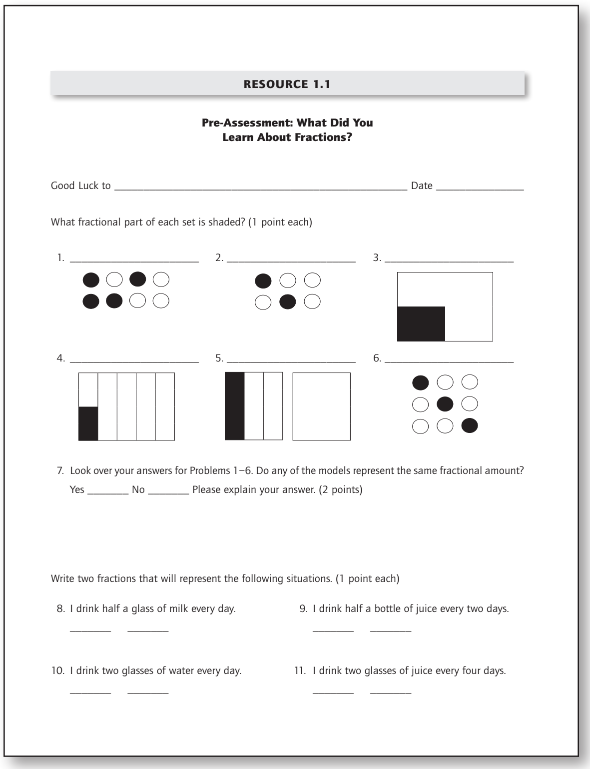
American Association for the Advancement of Science. (1993). *Benchmarks for science literacy*. Washington, DC: Author.

Clarke, D. M., Roche A., & Mitchell, A. (2008). 10 practical tips for making fractions come alive and make sense. *Mathematics Teaching in the Middle School*, *13*(7), 373–379.

Warrington, M. A. (1997). How children think about division with fractions. In D. L. Chambers (Ed.), *Putting research into practice in the elementary grades: Readings from* Journals of the National Council of Teachers of Mathematics (pp. 151–154). Reston, VA: National Council of Teachers of Mathematics.

RESOURCES

The following Resources can be found at the companion website for *Parallel Curriculum Units for Mathematics, Grades 6–12* at www.corwin.com/math6–12.



RESOURCE 1.1

Pre-Assessment: What Did You Learn About Fractions?

Good Luck to ______ Date ______

What fractional part of each set is shaded? (1 point each)

1. ______ 2. ______ 3. ______

4. ______ 5. ______ 6. ______

7. Look over your answers for Problems 1–6. Do any of the models represent the same fractional amount? Yes ______ No ______ Please explain your answer. (2 points)

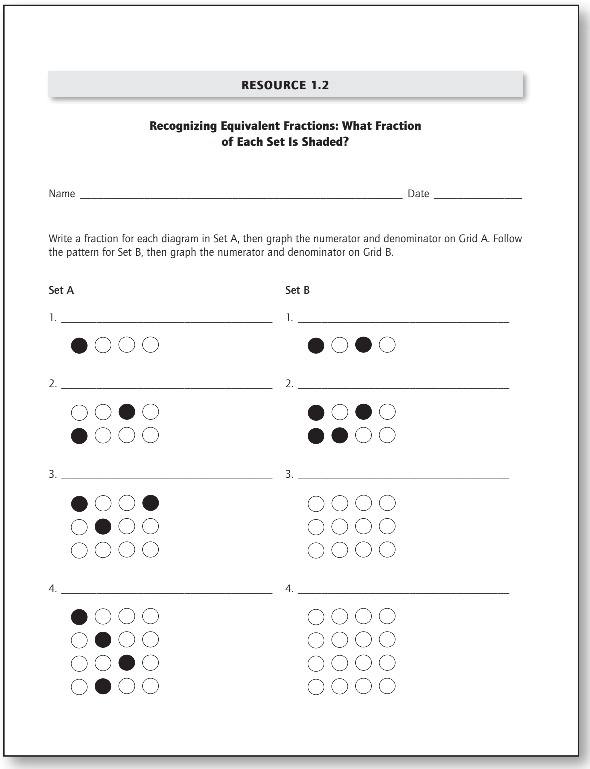
Write two fractions that will represent the following situations. (1 point each)

8. I drink half a glass of milk every day. ______ ______

9. I drink half a bottle of juice every two days. ______ ______

10. I drink two glasses of water every day. ______ ______

11. I drink two glasses of juice every four days. ______ ______



RESOURCE 1.2

Recognizing Equivalent Fractions: What Fraction of Each Set Is Shaded?

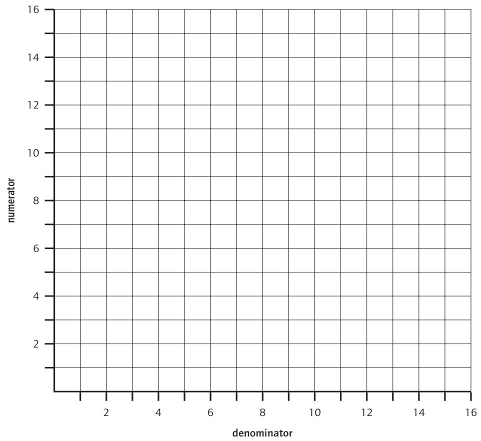
Name ______ Date ______

Write a fraction for each diagram in Set A, then graph the numerator and denominator on Grid A. Follow the pattern for Set B, then graph the numerator and denominator on Grid B.

Set A	Set B
1. ______	1. ______
2. ______	2. ______
3. ______	3. ______
4. ______	4. ______

RESOURCE 1.3

Overhead Template



RESOURCE 1.4

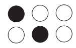
What Fraction Is Shaded?

Name ______________________________ Date __________

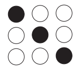
Set A

1. ____________

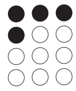
2. ____________



3. ____________



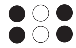
4. ____________



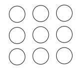
Set B

1. ____________

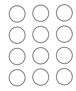
2. ____________



3. ____________



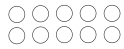
4. ____________



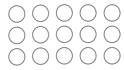
Set C: Shade $\frac{3}{5}$ and Graph

1. ____________

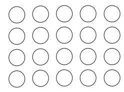
2. ____________



3. ____________



4. ____________



RESOURCE 1.5

Fractions and Rates of Change

Name ______________________________ Date __________

Part I

What is the simplest way to represent this rate of change?

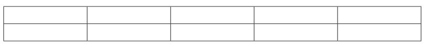
1. $\frac{3}{6}$ = __________
2. $\frac{3}{15}$ = __________
3. $\frac{35}{50}$ = __________
4. $\frac{27}{36}$ = __________
5. $\frac{12}{60}$ = __________
6. $\frac{7}{42}$ = __________

Part II

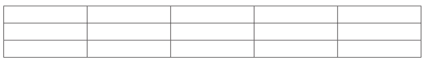
On each of the following four grids below, shade two-thirds of the rectangle strips to show equivalent fractions on each grid. Using a different color to represent each grid, plot your fractions on the graph and answer the questions below.

Grid 1

Grid 2



Grid 3

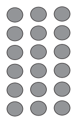


Grid 4

RESOURCE 1.6

Lemon Cookies

Name ______________________________ Date __________



Here are some lemon cookies. Look at the cookies to find the answer to each question.

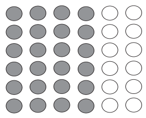
Example: Can you see thirds? How do you see them? How many cookies will you eat if you eat $\frac{2}{3}$ of the lemon cookies?

1. Can you see ninths? How do you see them? How many lemon cookies will you eat if you eat $\frac{4}{9}$ of the cookies?
2. Can you see sixths? How do you see them? How many lemon cookies will you eat if you eat $\frac{5}{6}$ of the cookies?
3. Can you see 36ths? How do you see them? How many lemon cookies will you eat if you eat $\frac{14}{36}$ of the cookies?
4. Can you see fourths? How do you see them? How many lemon cookies will you eat if you eat $\frac{3}{4}$ of the cookies?
5. Can you see 12ths? How do you see them? How many lemon cookies will you eat if you eat $\frac{5}{12}$ of the cookies?

RESOURCE 1.7

Our Favorites: Chocolate and Vanilla Cookies

Name ______________________________ Date ______________



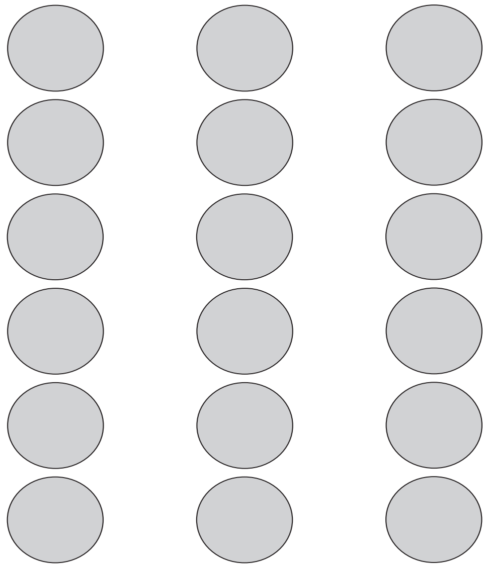
Here are some chocolate and vanilla cookies. Look at the cookies to find the answer to each question.

Example: If we put these cookies in six packs with an equal number of chocolate and vanilla cookies in each pack, how would you name the part of the cookies that is chocolate in each pack?

1. If we put the cookies in packs of four cookies each, what part of the cookies is chocolate?
2. If we wrap each cookie separately, how would you name the part of the cookies that is chocolate?
3. If we pack the cookies in dozens, how would you name the part of the cookies that is chocolate?
4. If we pack the cookies in packs of three cookies each, how would you name the part of the cookies that is chocolate?
5. The vanilla cookies fill four packages. Use packages of the same size to name the chocolate cookies.
6. The vanilla cookies fill $1\frac{1}{2}$ packages. Use packages of the same size to name the chocolate cookies.
7. The chocolate cookies fill $2\frac{2}{3}$ packages. Use packages of the same size to name the vanilla cookies.

RESOURCE 1.8

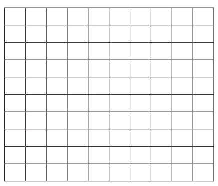
Cookies Template



RESOURCE 1.9

Tiling the Family Room

Name ______________________________ Date ______________



Here is the floor of your family room. Assume each square is 6 inches by 6 inches. You will create a pattern on the floor using the information that is provided below. It may be helpful for you to use a sheet of graph paper to show your patterns.

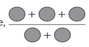
1. Your parents are going to let you use two colors for the floor. You have selected black and white. Each tile is 6 inches by 6 inches. They have asked that you put one color along the entire perimeter of the floor. You can make a pattern inside the square that remains using the two colors. You have decided to use black tiles for the perimeter.
   a. What fractional part of the floor is represented by the black tiles that you used for the perimeter?
   b. What fractional part of the floor remains?
   c. If your parents want one-fourth of the space that remains to match the color of the perimeter tiles, how many black tiles will you need for the square that remains?
   d. How many black tiles will you need for the entire floor?
   e. What fractional part of the entire floor is this?

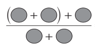
RESOURCE 1.10

Modeling Fractions

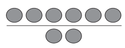
Name ______________________________ Date ______________

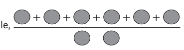
How many groups can be made? How much is left over? (The denominator gives the size of the groups.)

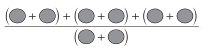
For example,  or $\frac{3}{2}$ can be represented as

 $= \frac{(1+1)+1}{(1+1)} = \frac{2+1}{2} = \frac{2}{2} + \frac{1}{2} = 1\frac{1}{2}$

This next example is slightly different, but the questions are the same as the ones above.
How many groups can be made? How much is left over?



For example,  or $\frac{6}{2}$ can be represented as:

 $= \frac{(1+1)+(1+1)+(1+1)}{(1+1)}$

$$\frac{2+2+2}{2} = \frac{2}{2} + \frac{2}{2} + \frac{2}{2} = \frac{3(2)}{2} = 3\left(\frac{2}{2}\right) = 3$$

What are the differences between the first example and the second example?

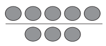
RESOURCE 1.11

Working With Fractions

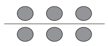
Name ______________________________ Date ____________

1. Use the methods just modeled to show the steps that will lead from the geometric model to the numerical representation.

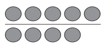
a.



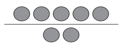
b.



c.



d.



2. Using what you discovered above, write the following fractions as mixed numbers in simplified form.

a. $\frac{5}{3} =$ b. $\frac{2+6}{2+3} =$ c. $\frac{2+(3+3)}{2+(3+2)} =$ d. $\frac{8}{5} =$

e. $\frac{2+(3\cdot 2)}{2+(3\cdot 1)} =$ f. $\frac{12}{10} =$ g. $\frac{2\cdot 6}{2\cdot 5} =$ h. $\frac{8}{7} =$

i. $\frac{2\cdot 5}{2\cdot 3} =$ j. $\frac{2+6}{2+5} =$ k. $\frac{2\cdot(3+3)}{2\cdot(3+2)} =$ l. $\frac{1+4}{1+2} =$

RESOURCE 1.12

Fractured Fractions and Beyond

Name ______________________________ Date ____________

Match each of the following problems with its equivalent values. Be prepared to explain your choices.

1. ________ $5\left(\frac{1}{4}\right)$ a. $\frac{1}{2}(5 \times 7)$

2. ________ $2\left(\frac{2}{5}\right)$ b. $\frac{1}{2}(x)$

3. ________ $\frac{35}{2}$ c. $\frac{4}{5}$

4. ________ $\frac{2}{5 \times 7}$ d. $2x\left(\frac{1}{x}\right)$

5. ________ $\frac{2}{x}$ e. $2\left(\frac{1}{5}\right)\left(\frac{1}{7}\right)$

6. ________ $\frac{x}{2}$ f. $\frac{bh}{2}$

7. ________ $\frac{1}{2}(2+3)$ g. $\frac{5}{4}$

8. ________ X h. $2\left(\frac{1}{x}\right)$

9. ________ 2 i. $2\left(\frac{x}{2}\right)$

10. ________ $\frac{1}{2}(bh)$ j. $\frac{2+3}{2}$

RESOURCE 1.13

Decomposing Fractions

Name ______________________________ Date ____________

Decompose the following problems by giving two alternative forms that will result in the same numerical answer. If you can simplify the fraction, then simplify it.

1. $\frac{5}{8} =$ ________ = ________ Simplify = ____________
2. $\frac{2+4}{3+4} =$ ________ = ________ Simplify = ____________
3. $\frac{3(1+1)}{2(1+3)} =$ ________ = ________ Simplify = ____________
4. $\frac{2}{5}(10) =$ ________ = ________ Simplify = ____________
5. $\frac{2}{5}\left(\frac{1}{10}\right) =$ ________ = ________ Simplify = ____________
6. $\frac{1}{3}(12r^2) =$ ________ = ________ Simplify = ____________
7. $(4+3)\left(\frac{x}{7}\right) =$ ________ = ________ Simplify = ____________

Write the following fractions as mixed numbers in simplified form.

8. $\frac{7}{3} =$ ____________ 9. $\frac{3+6}{3+3} =$ ____________

10. $\frac{4+(6+6)}{4+(6+4)} =$ ____________ 11. $\frac{9}{7} =$ ____________

12. $\frac{4+(6\times 4)}{4+(6\times 2)} =$ ____________ 13. $\frac{8}{6} =$ ____________

14. $\frac{4\times 6}{4\times 5} =$ ____________ 15. $\frac{3}{2} =$ ____________

16. $\frac{2\times 4}{2\times 3} =$ ____________ 17. $\frac{2+6}{2+5} =$ ____________

18. $\frac{2\times(3+3)}{2\times(3+2)} =$ ____________ 19. $\frac{1+4}{1+2} =$ ____________

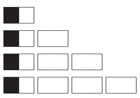
RESOURCE 1.14

Rectangle Pieces

Name ______________________________ Date ____________

Set A

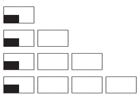
1.
2.
3.
4.



Number of Blocks	Shaded Part as a Complex Fraction	Simple Form for the Shaded Part	Unshaded Part as a Complex Fraction	Simple Form for the Unshaded Part	Sum of the Simple Shaded and the Simple Unshaded
1					
2					
3					
4					

Set B

1.
2.
3.
4.

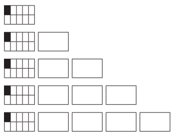


Number of Blocks	Shaded Part as a Complex Fraction	Simple Form for the Shaded Part	Unshaded Part as a Complex Fraction	Simple Form for the Unshaded Part	Sum of the Simple Shaded and the Simple Unshaded
1					
2					
3					
4					

RESOURCE 1.15

Rectangles Break Apart

Name ______________________________ Date ____________



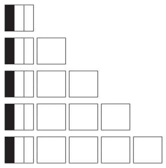
1. Complete the table below.

Number of Blocks	Shaded Part as a Complex Fraction	Simple Form for the Shaded Part	Unshaded Part as a Complex Fraction	Simple Form for the Unshaded Part	Sum of the Simple Shaded and the Simple Unshaded
1					
2					
3					
4					
5					
10					
20					
30					
40					
50					
100					

RESOURCE 1.16

Rectangles on the Move

Name ______________________________ Date ____________



1. Complete the table below.

Number of Blocks	Shaded Part as a Complex Fraction	Simple Form for the Shaded Part	Unshaded Part as a Complex Fraction	Simple Form for the Unshaded Part	Sum of the Simple Shaded and the Simple Unshaded
1					
2					
3					
4					
5					
10					
20					
30					
40					
50					
100					
N					

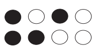
RESOURCE 1.17

Post-Assessment:
What Did You Learn About Fractions?

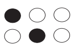
Good Luck to ______________________________ Date ____________

What fractional part of each set is shaded? (1 point each)

1. ________________

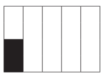


2. ________________

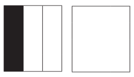


3. ________________

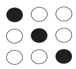
4. ________________



5. ________________



6. ________________



7. Look over your answers for Problems 1–6. Do any of the models represent the same fractional amount?
Yes ________ No ________ Please explain your answer. (2 points)

Write two fractions that will represent the following situations. (1 point each)

8. I drink half a glass of milk every day.
________ ________

9. I drink half a bottle of juice every two days.
________ ________

10. I drink two glasses of water every day.
________ ________

11. I drink two glasses of juice every four days.
________ ________