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Conjecture



One major drawback of having students spend their formative years memorizing facts is that facts change.

—Kelly Gallagher,
Veteran Educator and Author

Most kids show up for kindergarten already making routine use of higher-order thought processes. They don't need to be taught how to think. They need to learn how to examine, elaborate, and refine their ways of thinking and put this thinking to deliberate use converting information into knowledge, and knowledge into wisdom.

—Marion Brady, Teacher,
Education Administrator, and Author

THE POWER OF MYSTERIES

I am a fan of television crime dramas, shows like *CSI* and *NCIS*. Each week, I enjoy following along as the investigators gather the evidence and piece together the solution to a puzzling mystery. I try to anticipate where the story is going and to figure things out before the episode concludes. Sometimes, I'm able to work out who committed the crime, but the more satisfying stories contain a surprise twist which changes how I understand the puzzle.

Imagine if one week your favorite crime drama began with a rundown of all the clues, and then explained how they were all connected to the criminal. I know that I'd be pressing delete on the DVR pretty quickly. I mean, what's the point of watching the rest of the show if the mystery is already solved at the beginning?

This scenario is similar to what many students experience in a traditional math class. We, the teachers, lay everything out in front of them, and all they have to do is connect the dots in the way we already prescribed so that they can come up with the predetermined outcome. Only, in this show, we take away the remote, so the students no longer have the option to stop and skip to another show that's more interesting.

THE CURIOUS BRAIN

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There is a reason that mystery stories are enormously popular and have been since they first appeared in the early 1800s. Simply put, human brains are curious: we are wired to wonder. John Medina, molecular biologist, and author of *Brain Rules*, explains that it comes from our need to explore our environment.

Babies are born with a deep desire to understand the world around them, and an *incessant curiosity* that compels them to aggressively explore it. This need for explanation is so powerfully stitched into their experience that some scientists describe it as a drive, just as hunger and thirst and sex are drives. (Medina, 2014, p. 247, emphasis mine)

We love mysteries because our brains crave the joy that comes with discovery. Curiosity also improves learning, even for incidental or otherwise boring topics (Gruber, Gelman, & Ranganath, 2014).

For students to have this experience, they must have problems to solve. Though we may think our classrooms offer it, little of what normally takes place in a math classroom has anything to do with problems.

Real problem solvers pursue things they find puzzling instead of waiting for someone to present them with a problem. Psychologists call this *adopting a problem-solving orientation* and *searching through a problem space* (Malouff, n.d.; Newell & Simon, 1972).

Stop for a moment and consider your own experiences in math classes. How much of that experience involved adopting a problem-solving orientation and searching through a problem space? These are both dimensions of the first Principle: Conjecture.

Mathematics teacher David Wees tells a story about searching through his own problem space as a young man. Consider how you might react if a student shared this “wondering” in the middle of your lesson.

When I was 13, I remember discovering a really interesting relationship between numbers. I remember adding up $1 + 2 + 3$ and getting 6, and realizing that this was 2×3 . I then added up $1 + 2 + 3 + 4$ and got 10, which was $4 \times 5 \div 2$. Then I had an insight that 6 was also $3 \times 4 \div 2$, and that maybe there was some relationship between multiplying these numbers and getting the sum of the consecutive numbers. After a few minutes of playing around, I confirmed that if I took the last number I added and multiplied it by the next number, and then divided by 2, I always got the sum of all of the numbers.

Later that year I learned the algebraic way of representing this formula; $S = \frac{n(n+1)}{2}$. By figuring out the formula by myself, the algebra made much more sense.

Conjecture is both a habit of mind for a student and a cultural orientation within a classroom. It is a focus on questions rather than answers, and it is a sense that there is always more to figure out. In classrooms that honor conjecture, students not only question authority, they do so without fear of repercussions. We frequently hear “I wonder” and “what if,” and everyone challenges answers to all but the simplest questions. Conjecture is about options and possibilities, not about the one true path.



Problem-First

MP1 and MP3



Too often in math we lose the opportunity to take advantage of students’ natural curiosity when we take a linear approach to instruction. Take a look at any published math text, and find the problems in it. What patterns do you see? Content is likely broken into topic chapters, with each chapter

divided into eight to twelve daily lessons. The lessons probably begin with a demonstration of an isolated skill, followed by guided practice of the skill, then a collection of exercises to practice the skill independently. After the exercises, you find a few “application” problems, though in most cases they are really just exercises in disguise. If you’re lucky, after all of these exercises you might find a couple of “challenge” problems, usually marked with a star to indicate that they are only for the brave.

If you look at the structure of the chapter, you see a similar pattern. There are a series of skill-based lessons, and then at the end, right before the chapter test review there is one lonely lesson on problem solving. The thing is, it’s usually focused around teaching a specific strategy in the same way that the other skills were taught: modeling, guided practice, independent practice with exercises that focus narrowly on the specific strategy.

This format is based on an assumption I’ve heard many times: problem solving has to come only after all the basic skills are mastered. I have even seen entire course outlines where all of the problem solving is saved for the end of the year, after the state standardized testing is over. Think about what this does to a child’s curiosity. What’s more intriguing: a collection of skill drills, or a conundrum begging to be decrypted? Why, then, would

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we leave the intriguing part for last? If a mystery novel were laid out the way we teach math, it would start with all of the collected evidence and end with the crime. How many people would buy that book?

The cure is simple: put the problem first. Begin your unit of study with the starred problem. Pose a challenge so compelling that students are begging you to help them figure it out. At this point, students have opened the door for you to provide them with the tools and strategies they need. There are neurological reasons to do this. Our brains cannot attend to isolated facts and details. “Normally, if we don’t know the gist—the meaning—of information, we are unlikely to pay attention to its details. The brain selects meaning-laden information for further processing and leaves the rest alone” (Medina, 2014, p. 114).

Matt Bramucci did just that. The students in his high school music technology course were neither expert musicians nor problem solvers. In fact, most of them could not read music. One day, as the students arrived for what they thought was going to be an ordinary class, they found Mr. Bramucci standing and staring at a video playing on the classroom television. He appeared to be completely engrossed in the recording, which showed a house decked out in multicolored holiday lights, flashing in time to the Trans-Siberian Orchestra’s song “Wizard in Winter.”

“What are you doing, Mr. Bramucci?” one of the students asked.

“Shh!” Matt waved his hand impatiently at the student. “I’m trying to figure this out.” He continued to watch the video as it played. Students watched with him, and soon the entire class was completely transfixed. With a single whispered sentence, a teacher transformed a simple video into an intriguing challenge: “I wonder if we could do that?”

Students spent the next several weeks researching the technology and figuring out how to connect and program the hardware in order to create a similar light show. Along the way, other questions arose, such as “I wonder if we can make this work with live music, too?” New avenues of exploration opened up, and they learned more about music and the math behind it than they would have if Bramucci had marched them stepwise through the individual skills.

Of course, just planting a problem at the feet of your students is not enough on its own, but by starting with the problem, you provide context, motive, and opportunity for learning the content. To better understand the process that students use when they are solving problems, and to help you teach it better, let’s look at the related cognitive processes.

PROBLEM SOLVING IS A CYCLE, NOT A RECIPE

Yale University psychologists Jean Pretz, Adam Naples, and Robert Sternberg (2003) describe the problem-solving process as a cycle (pp. 3–4):

1. Recognize or identify the problem.
2. Define and represent the problem mentally.
3. Develop a solution strategy.
4. Organize his or her knowledge about the problem.
5. Allocate mental and physical resources for solving the problem.
6. Monitor his or her progress toward the goal.
7. Evaluate the solution for accuracy.

In a traditional mathematics classroom, the primary goal is for students to get the right answers to questions and exercises. Almost all of a student’s cognitive effort is focused on seeking answers, which takes place during steps 5 and 6. All other steps in the process have already been done for the student. Someone else, generally the teacher or textbook author, has identified and defined the problem, presented an efficient strategy, and organized all of the available information, prior to even presenting it to the

student. Even the last stage is ordinarily left to the teacher (or the student with an answer key) to verify the accuracy of the “solution.”

Medina (2014) puts this in perspective with an enlightening vignette:

If you could step back in time to one of the first Western-style universities, say, the University of Bologna, and visit its biology labs, you would laugh out loud. I would join you. By today’s standards, biological science in the 11th century was a joke, a mix of astrological influences, religious forces, dead animals, and rude-smelling chemical concoctions. But, if you went down the hall and peered inside Bologna’s standard lecture room, you wouldn’t feel as if you were in a museum. You would feel at home. There is a lectern for the teacher to hold forth, surrounded by chairs for the students to absorb whatever is being held forth—much like today’s classrooms. Could it be time for a change? (pp. 256–257)

While today’s mathematics classroom may not be as lecture-centered, it still consists for the most part of teachers presenting an algorithm, students absorbing that algorithm and then reproducing it until the algorithm becomes automatic. Repeat until graduation.

Often in today’s mathematics classroom, the focus is on either:

- Drilling of DOK Level 1 (Recall) facts and skills, or
- Rehearsing procedures for more complex problems to reduce them down to Level 1 tasks.

It is like a cooking class where students memorize a collection of recipes and techniques, but never go deeper. They never delve into the principles of flavor and texture, food science, or nutrition, and they never learn the reasons the techniques work.

In a classroom based on the 5 Principles, however, students ask most of the questions, and the answer to a question is very often another question.

Students pursue things they are curious about rather than things they are told to solve. The intrinsic joy of discovery, the essence of what it means to learn, is the centerpiece of all activity. In other words, when given a basket of mystery ingredients, students ought to understand enough about cooking to be able to make something tasty and satisfying without a memorized recipe.

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This is not to say that the teacher has no role or that students are left to pursue anything and everything that strikes their fancy. The teacher’s

role in a problem-solving culture is to find or create intriguing problems and to frame them in ways that students find compelling. Let's explore some of the strategies and mind-sets that you can establish to transform your classroom into one that promotes conjecture.

GRADES K–3

This chapter and the next four chapters contain five identified sections labeled like this one, highlighting strategies that are particularly useful in a specific grade band. These sections illustrate what the Principle looks like in those grades. All of the strategies, however, are applicable in any grade. If you are a high school teacher, for example, you can find excellent advice in this section that applies to your students as well. Likewise, the suggestions for upper grades can adapt well to elementary classrooms.

The Principle of Conjecture in the primary grades is all about maintaining and encouraging for as long and as deeply as possible students' natural curiosity about the world.



Never End With the Answer

MP3, MP6, and MP8



A Conjecture-oriented classroom expects students to think and reflect about their reasoning on every problem. Accordingly, with few exceptions no answer to any question should stand on its own in a math lesson. Even if it is correct, the teacher should follow the response with another question. In Grades K–3, I suggest these types of questions:

- Why do you think so?
- How do you know?
- How did you get that answer?
- Why did you solve it that way?
- Are there any other ways to answer it?
- What was hard about solving that problem?
- How did you overcome the difficulty?
- What did you use to help you solve this? How did it help?

At first, students will get annoyed or frustrated at constantly being challenged and asked “why,” but it won't be long before the mindset becomes ingrained and students start asking these questions of themselves and each other. Students learn that the “end” of a problem is really just the start of the next phase.

This is also a simple and effective way to add cognitive depth to a textbook exercise. An exchange in a second-grade classroom might follow this example. The students have all had a few minutes to work on this problem:

Lee brought 8 cupcakes to the party. Maria brought 9 cupcakes. Each person at the party ate one cupcake. After the party was over, there were still 3 cupcakes left. How many people were at the party?

Teacher: “OK, so who can answer this question?”

Six hands are raised immediately. The teacher waits a few more seconds while several other hands appear.

Teacher: “Mark?”

Mark: “Twenty?”

Teacher: “You sound unsure. How do you know it’s twenty?”

Mark: “Because that’s the answer I got when I added.”

Teacher: “So why did you solve it that way?”

Mark: “I took all the numbers and added them.”

Teacher: “OK, but why does it make sense to add them all?”

Mark: “I don’t know.”

Teacher: “Can anyone else help explain? Anita?”

Anita: “Well, two different people brought cupcakes and put them together, so you have to add.”

Teacher: “OK, tell me more.”

Anita: “Uh, but I don’t know what to do with the three that are left at the end. I don’t think you’re supposed to add those.”

Teacher: “Why not? Why doesn’t that make sense to you?”

Anita: “Well they were left over. Doesn’t that mean we should subtract them?”

Teacher: “Mark, what do you think?”

Mark: “Now I’m not sure.”

Teacher: “OK, so what seems hard about this to you?”

Mark: “I thought I was supposed to add the ones that were left, but Anita says we should subtract.”

Teacher: “So, it sounds like you’re trying to figure out what operation to use with the numbers in the problem. Is there another way you could figure out what’s going on?”

Mark: “Well, maybe if we had a party with some cupcakes.”

Laughter in the classroom.

Teacher

(laughing also): “That’s an interesting idea. Is there a way we could do that?”

Mark

(surprised): “Really? We can have a party?”

Teacher: “No, we’re not going to have a party, but how could we use that idea to help solve the problem?”

Anita: “Maybe we could use pretend cupcakes? Can I be Maria?”

Teacher: “Sure. Mark, would you like to act out the part of Lee?”

Mark: “Yes!”

Notice at no time did the teacher indicate whether any answer or potential solution was right or wrong. He or she did not use leading questions that cue the correct response, either, but very deliberately left the solution path and the answer open to discussion as long as possible.

While this approach helps students increase their ability to reason and discuss math, be aware that it’s still very teacher centric. It does allow for increased depth, but there are still only a few students involved in the discussion.

Digital Tools and Resources for Conjecture in K–3

Each grade band in this book includes a brief section describing one or more digital tools that can support the Principle and help you create a problem-solving culture within your classroom. These sections do not give detailed tutorials. I just want to point you in the direction of tools that can strengthen your transformation plan with a few quick ideas for incorporating them into your teaching. Then at the end of the chapter, I highlight one or two tools mentioned in the chapter with a more extended example showing how teachers have integrated them into a 5 Principles classroom.

Part of the goal of the “Never End With the Answer” strategy is to encourage young children to verbalize their reasoning. To extend this idea, try using digital whiteboard apps to allow students to narrate their problem solutions.

One example of this is **Educreations** (<http://www.educreations.com>). At this site, either using a computer web browser, or on an iPad, teachers and students can use a simple whiteboard to draw and write while recording

verbal narration. Even young children who are intimidated by the idea of writing out their math thinking can talk it out by recording. Recordings are private, and only the teacher can access them.

Other similar products are **ShowMe** (<http://www.showme.com>), which is exclusively available on iPad, and **PixiClip** (<http://www.pixiclip.com>), which is a web-only application. If you want more flexibility, try using screen casting software, such as **Screencast.com** or **Screencast-O-Matic.com**. Students can use these tools to record everything they are doing on their computer screens as well as voiceover activity. These products probably require more assistance from you to make them work, but they allow the student to use any software on the computer and aren't limited to the specific tools embedded in the whiteboard apps.

Students can share their recordings, and you can use student made videos to prompt inquiry by showing the beginning of a new problem, allowing the class to work on solving it, then showing the student's solution.

GRADES 2-5

In Grades 2-5, the focus shifts from simply nurturing inquiry to guiding and organizing it. Students need help to turn raw curiosity into productive thinking, and the strategies here are designed to achieve that while still encouraging students to continue wondering freely.



Always Ask Why

MP3



As teachers, we are accustomed to asking questions when we already know the answers, and to keep calling on students until we get the answer we're expecting. At that point, we move on to the next question with no further thought.

To create a culture of inquiry and the habits of mind required by the Common Core, expect students to provide explanations of their reasoning. Ask the students their thoughts on it. Ask how they know the respondent is correct. You can also turn some of the responsibility for probing and asking follow-up questions to the students.

Keep students thinking and questioning. This is even more powerful when it becomes the default process for all student responses.

Keep students thinking and questioning, and encourage reasoning to move far beyond merely guessing. This is even more powerful when it becomes the default process for all student responses. You will notice a predictable

pattern when you shift to this strategy. At first, students are thrown off, and many stop volunteering as much, since they are being held accountable for their thinking rather than being able to throw an answer out into the wild and hope for the best. Don't panic; stick with it. Eventually, students gain confidence in their ability to answer, especially if you give them time to think and to share in pairs or small groups before (or instead of) large group responses.

Asking students *why* is the first step away from pure guesswork into the world of metacognition. In Singapore, often held up as the gold standard of mathematics education, metacognition is one key to excellent learning in math. Dr. Wong Khoon Yoong, a mathematician and educator from that country, discusses the importance of metacognition to problem solving:

The Wild Goose Chase in Problem Solving

When solving standard mathematics problems, students normally recall and apply learned procedures in a straightforward way. However, if the problem is unfamiliar, some students simply pick a method and keep persistently on the same track for a long time without getting anywhere. Schoenfeld (1987) described this behaviour as chasing the wild mathematical goose. A different behaviour is also observed: some students jump from one rule to another in a haphazard way hoping to find the correct answer, become agitated, frustrated and finally give up. Teachers who observe both types of unsuccessful behaviours may think that the students have not mastered the skills, and then proceed to re-teach the skills. . . . A more "metacognitive" teacher believes that the students' difficulty may not be with the skills, rather it might indicate a lacking in self-regulation of the problem-solving process. (Wong, 2002, pp. 1–2)

Metacognitive Questions

Lisa Chesser (2014) at the TeachThought blog created a comprehensive list of questions that promote metacognition. Here, I've adapted a few that are particularly helpful for the mathematics classroom in Grades 2–5. Note how these questions promote deeper reflection than those in the last section:

1. What about this problem feels familiar? Why?
2. Why do you think this works? Does it always work? Why do you think so?
3. What about the strategy is working for you? What isn't working? Why?

4. Are there any other similar answers you can think of with alternative routes?
5. What patterns might lead you to an alternative answer?
6. Does anyone in this class want to add something to the solution?

Digital Tools and Resources for Conjecture in 2–5

In Grades 2–5, introduce problems that require lots of messy inquiry in order to help students break away from reliance on right answers and someone else checking their work. In these grades, students also have the perception that all problems have already been solved, and their job is simply to find (or guess) the answer that someone else already got; as a result, many kids ask why they should bother solving this if someone else already did it.

This is a great time to introduce the idea that there are mathematical problems out there which no one, not even professional mathematicians, has solved yet. While most of these are fairly obscure or esoteric, the outstanding website **MathPickle** (<http://www.mathpickle.com>) translates these unsolved problems into kid-friendly, scaled down versions. Head to the “Unsolved Problems for K–12” section (<http://mathpickle.com/unsolved-k-12>) to find video and downloadable resources for trying these problems in your classroom.

GRADES 4–7

Moving into the intermediate grades, students are ready for deeper analysis and more complex problems. Though the strategies in this section are useful in the primary grades and I encourage teachers of those grades to think about ways to adapt them, they pay significant dividends in student learning in upper elementary and middle school.



Pattern Watch

MP4, MP7, and MP8



The human brain is a natural pattern-finding machine. It is so good at patterns that we often create them where none really exist. It’s why we see constellations in the stars, and why tragedies *always* occur in threes.

One of the characteristics of good problem solvers is that they readily recognize and understand patterns, and that they distinguish real patterns